Integration of the Nonlinear Modified Korteveg-De Fries Equation Loaded in The Class of Periodic Functions

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Abstract
A method for solving the Kaup system A number of problems of mathematical physics are reduced to finding the eigenvalues and orthonormalized eigenfunctions of the Sturm-Liouville operator. In particular, when solving the equations of string vibration and heat conduction, which are considered the main equations of classical mathematical physics, by Fourier method, it is necessary to determine the eigenvalues of the Sturm-Liouville boundary value problem, orthonormalized eigenfunctions and expand the arbitrary function into the Fourier series using them.

Keywords: Class of periodic functions, antiperiodic problem, periodic problem, eigenvalues, Kaup system.

Introduction

INTEGRATION OF THE LOADED NONLINEAR MODIFIED KORTEVEG-DE FRIES EQUATION

Let's look at the following mKdf equation

$$q_t = 6q^2 q_x - q_{xxx}, \quad t > 0, \quad x \in R \tag{1}$$

$$q(x,t)\big|_{t=0} = q_0(x), \tag{2}$$

x according to π - is periodic and this

$$q(x,t) \in C_x^3(t>0) \cap C_t^1(t>0) \cap C(t \geq 0), \tag{3}$$

which satisfies the smoothness condition $q(x,t)$ be required to find the actual function:

$$q(x+\pi,t) \equiv q(x,t), \quad x \in R, \; t > 0$$

in this $q_0(x) \in C^3(R)$ is a given real function.

Теорема 1. If $q(x,t)$ - (1) - (2) be the solution to the problem. Then the Dirac operator λ_n, $n \in Z$ spectrum limits

$$L(\tau,t)y \equiv B\frac{dy}{dx} + \Omega(x+\tau,t)y = \lambda y, \quad x \in R, \tag{4}$$

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will be, in this

$$B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix},\ \Omega(x,t)=\begin{pmatrix}0 & q(x,t)\\ q(x,t) & 0\end{pmatrix},\ y=\begin{pmatrix}y_1(x)\\ y_2(x)\end{pmatrix},$$

τ and t will not depend on parameters, $\xi_n(\tau,t)$, $n\in Z$ and the spectral parameters are these

$$\frac{\partial\xi_n}{\partial t}=2(-1)^n\sigma_n(\tau,t)h_n(\xi)\cdot\{-2\xi_n[q^2(\tau,t)+q_x(\tau,t)]-4\xi_n^3\}, \qquad (5)$$

Dubrovin satisfies the analogue of the system of equations, where

$$h_n(\xi)=\sqrt{(\xi_n-\lambda_{2n-1})(\lambda_{2n}-\xi_n)}\cdot\sqrt{\prod_{\substack{k=-\infty,\\ k\neq n}}^{\infty}\frac{(\lambda_{2k-1}-\xi_n)(\lambda_{2k}-\xi_n)}{(\xi_k-\xi_n)^2}}.$$

Here $\sigma_n(\tau,t)$'s hint $\xi_n(\tau,t)$ the point is its own $[\lambda_{2n-1},\lambda_{2n}]$ lacunae changes to its opposite at each encounter with the border. In addition, the following prerequisites are met

$$\xi_n(\tau,t)\big|_{t=0}=\xi_n^0(\tau),\quad \sigma_n(\tau,t)\big|_{t=0}=\sigma_n^0(\tau),\quad n\in Z, \qquad (6)$$

Here $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$ $n\in Z$ – $p_0(x+\tau)\equiv 0$, $q_0(x+\tau)$ spectral parameters of the Dirac operator with coefficients.

Proof. (4) for eq

$$y_1(0)=0,\ y_1(\pi)=0,$$

of the Dirichlet problem $\xi_n=\xi_n(\tau,t)$, $n\in Z$ orthonormalized eigenvector-functions corresponding to eigenvalues $y_n=\left(y_{n,1}(x,\tau,t),\, y_{n,2}(x,\tau,t)\right)^T$, $n\in Z$ we define through $(L(\tau,t)y_n,y_n)=\xi_n$ equality t differentiated according to and $L(\tau,t)$ using the symmetry of the operator

$$\dot{\xi}_n=(\dot{\Omega}(x+\tau,t)y_n,y_n), \qquad (7)$$

we will have Do not do scalar multiplication

$$(y,z)=\int_0^{\pi}[y_1(x)\bar{z}_1(x)+y_2(x)\bar{z}_2(x)]dx,\quad y=\begin{pmatrix}y_1(x)\\ y_2(x)\end{pmatrix},\ z=\begin{pmatrix}z_1(x)\\ z_2(x)\end{pmatrix},$$

(7) using the following equation

$$\dot{\xi}_n=2\int_0^{\pi}y_{n,1}y_{n,2}\,q_t(x+\tau,t)dx,$$

we will write it down.

This

$$q_t(x+\tau,t)=6q^2(x+\tau,t)q_x(x+\tau,t)-q_{xxx}(x+\tau,t),$$

from equality

$$\dot{\xi}_n=2\int_0^{\pi}y_{n,1}y_{n,2}(6q^2q_x-q_{xxx})dx, \qquad (8)$$

it turns out to be. By calculating the derivative directly

$$\{2\xi_n q_x \cdot (y_{n,1}^2 - y_{n,2}^2) + 2(4\xi_n^2 q + 2q^3 - q_{xx})y_{n,1}y_{n,2} - 2\xi_n(2\xi_n^2 + q^2)\cdot(y_{n,1}^2 + y_{n,2}^2)\}'_x =$$
$$= 2y_{n,1}y_{n,2}(6q^2 q_x - q_{xxx}),$$

can be shown to be

Also

$$2\int_0^\pi y_{n,1}y_{n,2}(6q^2q_x - q_{xxx})dx =$$
$$= -2\xi_n[q^2(\tau,t) + q_x(\tau,t) + 2\xi_n^2]\cdot[y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t)], \qquad (9)$$

will be. (8) and (9) from

$$\dot{\xi}_n = [y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t)] \times \{-2\xi_n[q^2(\tau,t) + q_x(\tau,t)] - 4\xi_n^3\}, \qquad (10)$$

originates. of this equation $s_1(0,\lambda,\tau,t) = 0$, $s_2(0,\lambda,\tau,t) = 1$ a solution that satisfies the initial conditions $s(x,\lambda,\tau,t)$ we define through.

In that case

$$y_n(x,\tau,t) = \frac{1}{c_n(\tau,t)} s(x,\xi_n,\tau,t),$$

and in this

$$c_n^2(\tau,t) = \int_0^\pi [s_1^2(x,\xi_n,\tau,t) + s_2^2(x,\xi_n,\tau,t)]dx = -\frac{\partial s_1(\pi,\xi_n,\tau,t)}{\partial\lambda}\cdot s_2(\pi,\xi_n,\tau,t),$$

using these equations,

$$y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t) = \frac{s_2^2(\pi,\xi_n,\tau,t) - 1}{c_n^2(\tau,t)} = -\frac{s_2(\pi,\xi_n,\tau,t) - \dfrac{1}{s_2(\pi,\xi_n,\tau,t)}}{\dfrac{\partial s_1(\pi,\xi_n,\tau,t)}{\partial\lambda}}, \qquad (11)$$

we generate .

Here

$$s_2(\pi,\xi_n,\tau,t) - \frac{1}{s_2(\pi,\xi_n,\tau,t)} = \sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n) - 4}, \qquad (12)$$

putting the following expression

$$y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t) = -\frac{\sigma_n(\tau,t)\sqrt{\Delta^2(\xi_n) - 4}}{\dfrac{\partial s_1(\pi,\xi_n,\tau,t)}{\partial\lambda}}, \qquad (13)$$

we find that eq.

Using the following spreads

$$\Delta^2(\lambda) - 4 = -4\pi^2 \prod_{k=-\infty}^{\infty} \frac{(\lambda - \lambda_{2k-1})(\lambda - \lambda_{2k})}{a_k^2}, \quad s_1(\pi,\lambda,\tau,t) = \pi \prod_{k=-\infty}^{\infty} \frac{\xi_k - \lambda}{a_k},$$

in this $a_0 = 1$ ва $a_k = k$, $k \neq 0$, he above equation

$$y_{n,2}^2(\pi,\tau,t) - y_{n,2}^2(0,\tau,t) = 2(-1)^n \sigma_n(\tau,t) h_n(\xi), \quad (14)$$

we will write it down. (14) expression (10) putting it into equality, we get (5).

If the Dirichlet boundary conditions are periodic $y(\pi) = y(0)$ or antiperiodic $y(\pi) = -y(0)$ If we replace with boundary conditions, then (3.5) instead of eq $\dot{\lambda}_n = 0$ we generate .

So, periodic and antiperiodic problem λ_n, $n \in Z$ eigenvalues t will not depend on the parameter. **The theorem is proved.**

Result 1. This

$$q^2(\tau,t) + q_\tau(\tau,t) = \sum_{k=-\infty}^{\infty} \left(\frac{\lambda_{2k-1}^2 + \lambda_{2k}^2}{2} - \xi_k^2(\tau,t) \right), \quad (15)$$

taking into account the trace formula (5), the system can be recorded in a closed form.

Corollary 2. This theorem (1)-(3) gives a way to solve the problem. for this initially $q_0(x+\tau)$ corresponding to the coefficient λ_n, $\xi_n^0(\tau)$, $\sigma_n^0(\tau)$, $n \in Z$ we can find the spectral data. Then (5) for the system of Dubrovin equations

$$\xi_n(\tau,t)\big|_{t=0} = \xi_n^0(\tau), \ \ \sigma_n(\tau,t)\big|_{t=0} = \sigma_n^0(\tau), \ \ n \in Z$$

Let's solve the Koshi problem. After that

$$q(\tau,t) = \sum_{k=-\infty}^{\infty} (-1)^{k-1} \sigma_k(\tau,t) h_k(\xi(\tau,t)). \quad (16)$$

according to the trace formula $q(x,t)$ we find.

Conclusion:

Using the inverse spectral problem for the quadratic set of Sturm-Liouville operators with periodic coefficients, the Cauchy problem for Kaup's system of loaded terms is solved in the class of periodic functions. applied to finding solutions of the periodic class.

Adabiyotlar

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