

SOME INTERPRETATIONS OF LOBACHEVSKY'S GEOMETRY

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Abstract

The article deals with some interpretations of Lobachevsky's geometry.

Keywords: Lobachevsky, geometry.

LOBACHEVSKIY GEOMETRIYASINING BA'ZI TALQINLARI

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Annotatsiya:

Maqolada Lobachevskiy geometriyasining ba'zi talqinlari haqida gap borgan.

Kalit so'zlar: Lobachevskiy, geometriya.

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"Lobachevskiy geometriyasida
yo'llarni klassifikatsiyalash"
mavzusidagi dissertatsiya ishidan
olingan ayrim qismlari*

Reja:

1. Kirish. Lobachevskiy geometriyasi tarixi.
2. Asosiy Qism. Lobachevskiy geometriyasi haqida.
3. Xulosa. Lobachevskiy geometriyasining hayotimizga tatbiqi

Introduction

1.1 Historical overview of Lobachevsky geometry.

The axomatic construction of geometry was systematically introduced in the 3rd century BC through Euclid's Foundations. By the second century BC, the axiomatic construction of geometry was completed based on Euclid's thirteen-volume "Principles". As a result of practical needs, the measurement of angle, length and surface has found its confirmation with the help of consistent mathematical theories, axioms, postulates, theorems, definitions and proofs. Concepts of points, straight lines, and planes emerged as basic elements of geometry. At that time, there was no question of what expression a straight line of infinite distance would have. Perhaps that is why postulate V of parallelism was very carefully stated by Euclid: if a straight line intersects two straight lines, these straight lines intersect on the side where the sum of the interior one-sided angles is less than the sum of the two right angles.

The ancients imagined the earth as a flat disc of enormous size (which could not be measured). As a result of geographical discoveries, these border distances have been further extended. With the increase of knowledge about the surrounding existence, geometry also began to develop. Attempts to prove Euclid's famous postulate about parallels as a theorem, attempts to derive a conclusion from it by accepting the converse of the postulate led to the creation of non-Euclidean geometries.

The thinkers of eastern countries in the emergence of New Euclidean geometry were al-Jawhari from Arabia (9th century), Sabit ibn Korra from Baghdad (9th century), Iranian mathematician Abul Abbas al-Fadla Ibn Khatima (Samarkand, Bukhara, Israel, Marv), al-Nairizi (10th century), Omar Khayyam (11th century, 1048-1131), Muhyiddin al-Maghribi from Marogha (Azerbaijan) (13th century), Ibn Sina (980-1037), Ibn al-Haysam from Egypt (965-1039), Husam ad-Din as-Samr (XIII century), Azerbaijan was founded by Nasir ad-Din Tusi (XIII century) and others. In particular, the scientific works on geometry in the activity of Central Asian thinker Omar Khayyam, Ibn Sina and others became important and they became the founders. In particular, we can confirm that N. I. Lobachevsky was based on predictions (hypotheses) of Omar Khayyam when creating non-Euclidean geometry. Because Omar Khayyam predicted in his work: "The sum of the internal angles of a triangle cannot exceed 180° ." Later, this prediction (hypothesis) was confirmed in the geometry created by N. I. Lobachevsky as "The sum of the internal angles of a triangle is less than 180° " and it became one of the main theorems [14].

N. I. Lobachevsky was born on December 2, 1792 in Nizhny Novgorod (now Gorky). He graduated from the gymnasium at Kazan University, then Kazan University, where he was employed as a teacher. He worked as a professor in 1816 and rector of this university from 1827 to 1846. N. I. Lobachevsky died on February 24, 1856. Since working at this university, N. I. Lobachevsky tried hard to prove postulate V. He came to the conclusion that the efforts of his comrades were ineffective, that it was not enough to use the previous postulates to prove postulate V. To prove this, while maintaining Euclid's main directions, V rejected the postulate and replaced it with its opposite, building a logical system. This logical scheme led to the conclusion that a new geometry, such as Euclidean geometry, would succeed.

On February 7, 1826, N.I. Lobachevskiy delivered his lecture "On the Rules of Geometry" to the Faculty of Physics and Mathematics of Kazan University. In 1829, he included his article "On the Beginning of Geometry" in the "Works of Scientists of Kazan University" series. This was his first work on new geometry. In the following years, N.I. Lobachevsky studied a lot about geometry. In these works, he justified and clearly defined postulate V that it cannot be deduced from the rest of Euclid's axioms [9].

Lobachevsky improved his geometry by introducing trigonometric formulas in the plane and in space. He called this geometry "imaginary geometry".

Lobachevsky did not find a logical contradiction in his geometry while discovering new evidence. Wanting to show that this geometry never leads to contradictions, Lobachevsky conducts analytical checks on his geometry and solves the problem of contradiction.

Almost all of Lobachevsky's contemporaries believed that his geometry was flawed. They believed that not only this geometry is not used in the external world, but that during the further development of this geometry, an internal contradiction will arise in itself. For example, in



those difficult times, the mathematician Ostrogradsky (together with his students) did not understand and, as a result, gave a negative review of Lobachevsky's discovery. Among the mathematicians of that time, Gauss hesitated, that is, Lobachevsky refrained from expressing any opinion about the discovery of non-Euclidean geometry. Laplace, Lagrange, and Bunyakovsky retreated from the complexity of understanding the theory of parallels and Lobachevsky's non-Euclidean geometry. They were also involved in the theory of parallel lines. Later, after A. Poincaré, F. Klein, Beltrami gave interpretations of Lobachevsky's geometry, this discovery became clear to everyone. In order to understand and imagine Lobachevsky's geometry, we need to know Euclidean geometry and its concepts, axioms, and some information.

§ 1.2. Information about Lobachevsky geometry

Lobachevsky recommended considering Euclidean geometry on a cosmic scale. Omar Khayyam saw this first. The Earth, Sun and Sirius stars were taken as the points of the triangle to look at the cosmic scale. The famous French mathematician Jules Henri Poincaré (1854–1912) created the model of Lobachevsky geometry in 1882 and described it as follows: We take Euclidean geometry and draw a horizontal straight line in it, which divides the plane into two half-planes. We call the points of the upper half-plane non-Euclidean points (not including the points of a straight line). Semicircles whose centers lie on a straight line are called Euclidean straight lines. We also include rays perpendicular to the straight line in the Euclidean straight lines.

Earlier, in 1871, the German mathematician Felix Christian Klein (1849–1925) gave a description of Lobachevskii geometry based on the idea of the projective metric. In his 1872 "Comparative View of New Geometric Researches" (Erlangen Program), he presented his geometrical studies as a theory of invariants of any special group of geometrical permutations. You can move from one type of geometry to another by expanding or collapsing the group. Euclidean geometry is the science of invariants of metric groups; projective geometry is the science of invariants of projective groups. Classifying a group of permutations leads to classifying geometries. The theory of algebraic and differential invariants of each group gives the analytical structure of geometry. Later, Klein shows that Euclidean geometry can be modeled by projective metrics. In doing so, he was based on the concept of projective metric introduced in 1859 by the English mathematician Arthur Kelly (1821–1895). Lobachevsky hypothesized that if real space does not obey the laws of Euclidean geometry, then the sum of the angles of a triangle in space is less than 180° . He believed in non-Euclidean space. If we look at the visible part of the universe as a reduction, then Lobachevsky's geometry will be appropriate. In 1863, the Italian mathematician Eugenio Beltrami (1835-1900) and the German mathematician Bernhard Riemann (1826-1866) did great work on the description of the new geometry. Beltrami showed the existence of real bodies whose surfaces are filled with Lobachevsky geometry. He proved that non-Euclidean geometry holds on surfaces with constant negative curvature (pseudosphere), and also showed that Lobachevsky geometry is without logical contradiction.

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