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THE RATIONAL - SYSTEM WITH THE LIMIT SET CONSISTING CONNECTIVITY COMPONENTS

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Abstract
Studying the structure of a limit set is crucial for characterizing the long-term behavior and stability of a dynamical system. It is known that a bounded limit set is a continuum i.e. connected and compact, whereas unbounded ones may have enough complicated structure [1-11]. For instance, the limit set of -system may consist of uncountable connectivity components. It can be shown that -limit set of quadratic systems is always connected. There is a cubic system on the plane which possesses the -limit set consisting of two straight lines. The limit set of a polynomial system on the plane may have connectivity components for arbitrary large [2]. In this paper we consider a problem how to construct a rational dynamical system with the -limit set consisting connectivity components.

Keywords: dynamical system, rational system, vector field, limit set, Hamiltonian system, Hamiltonian function, connectivity components, unboundedness.

Introduction

To construct desired system firstly, we take following rational function

$$f^0(x,y)=\left(R^2k^2+x^2(y^2-k^2)\right)\cdot$$

$$\cdot\frac{\left(R^2k^2+\frac{(x-\sqrt{3}y)^2}{4}\left(\frac{(x\sqrt{3}+y)^2}{4}-k^2\right)\right)\left(R^2k^2+\frac{(x+\sqrt{3}y)^2}{4}\left(\frac{(x\sqrt{3}-y)^2}{4}-k^2\right)\right)}{x^{16}+y^{16}+R^6k^6},$$

where R and k are positive parameters. The level line $f^0(x,y)=0$ consists of six brunches $G_1,G_2,G_3,G_4,G_5,G_6,$ which divide $\square^2$ into seven connectivity components (see Figure 1).

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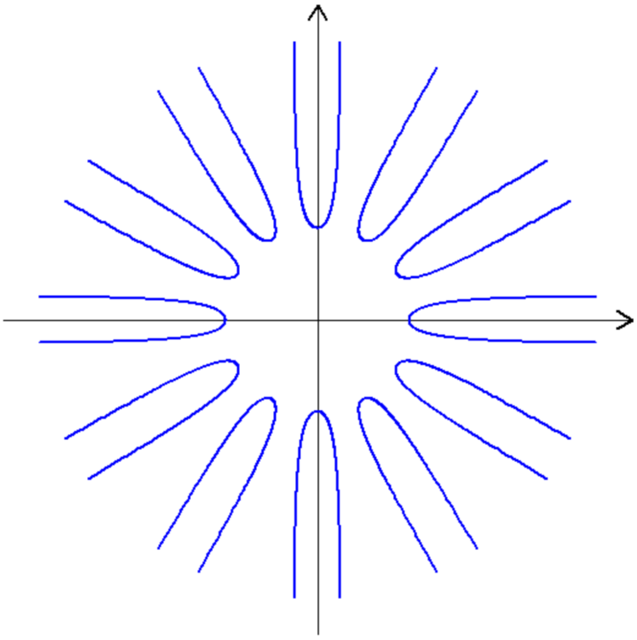

Figure 1. The level line $f^0(x,y)=0$.

Note that the value of parameter R is equal to the distance from angle of branches to the origin, $2k$ – the distance between parallel asymptotes of brunches.

Let n be an arbitrary natural number. Put $k = Rtg\frac{\pi}{6n}$ and define the function

$$f^m(x,y) = f^0\left(x\cos\frac{\pi m}{3n} - y\sin\frac{\pi m}{3n},\ x\sin\frac{\pi m}{3n} + y\cos\frac{\pi m}{3n}\right)$$

for $m = 1,2,...n-1$. Further, we consider the rational function

$$F(x,y) = f^0(x,y)\cdot f^1(x,y)\cdot ...\cdot f^{n-1}(x,y)$$

(the degree of numerator is equal to 12^n, the degree of denominator to 14^n). The level line $F(x,y)=0$ consists of level line G_1,G_2,G_3,G_4,G_5,G_6 and as well as lines obtained by rotating G_1,G_2,G_3,G_4,G_5,G_6 consequently through the angles $\frac{2\pi}{n}k,\ \ k=1,2,...,n-1$.

Thus the line $F(x,y)=0$ consists of $6n$ brunches.

Secondly, we construct the Hamiltonian system for which $F(x,y)$ serves the energy function:

$$\dot{x} = \frac{\partial F}{\partial y},\quad \dot{y} = -\frac{\partial F}{\partial x} \qquad (1)$$

where

$$F_x = \frac{\partial F}{\partial x} = F\sum_{i=0}^{n-1}\frac{f_x^i(x,y)}{f^i(x,y)},\qquad F_y = \frac{\partial F}{\partial y} = F\sum_{i=0}^{n-1}\frac{f_y^i(x,y)}{f^i(x,y)}.$$

The function $F(x,y)$ reaches its maximum, equal to 1, only at the point $(0,0)$. Moreover, $F(x,y) \to 0$ as $x^2 + y^2 \to \infty$. Thus the level lines $F(x,y) = c$ for $0 < c < 1$ are closed curves and they fulfills the region $0 < F(x,y) < 1$.

Finally, we modify the system (1) perturbing in the direction of the vector (F_x, F_y) such that the line $F(x,y) = 0$ stays an integral:

$$\dot{x} = F_y(x,y) + \lambda F(x,y) F_x(x,y)$$
$$\dot{y} = -F_x(x,y) + \lambda F(x,y) F_y(x,y), \qquad (2)$$

where λ is an enough small positive number.

Theorem. *The ω-limit set of trajectories of the system* (2), *lying in the region* $F(x,y) > 0$, *consists of the line* $F(x,y) = 0$ *and therefore, has* $6n$ *connectivity components* (see Figure 2).

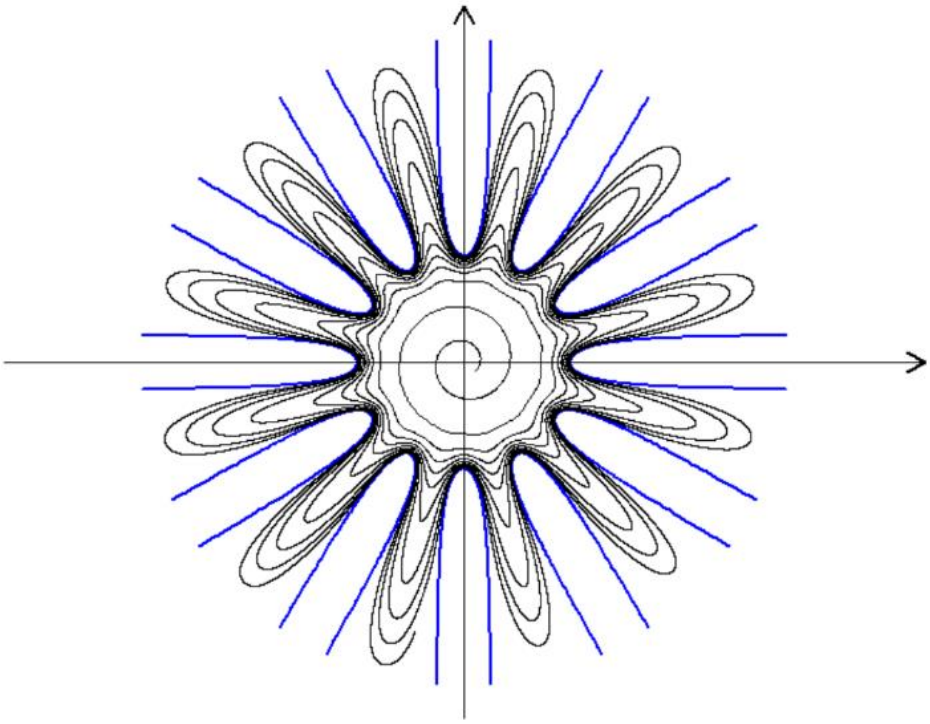

Figure 2. The level line $F(x,y) = 0$ and one of the trajectories when $n = 2$.

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