TO THE THEORY OF CURRENT AND VOLTAGE RESONANCE IN AN RLC OSCILLATOR CIRCUIT

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Abstract
This article is devoted to current and voltage resonances in RLC oscillatory circuits. Particular attention is paid to the theoretical foundations of resonant processes.

Keywords: Resonance, oscillatory circuit, charge resonance, current resonance, voltage resonance, quality factor, differential equation, driving force, damped oscillation, damping coefficient, vector diagram.

Introduction

If the value of the oscillatory circuit is great in the formation, or more precisely, in the generation of electromagnetic oscillations and waves, then the value of the resonance phenomenon in the oscillatory circuit is infinite in the transmission of electromagnetic waves into space or the capture and use of electromagnetic waves. This is the basis of the principle of radio transmission and reception [1]. Based on this, thanks to this article we will get acquainted with the theory of resonances of current and voltage in RLC oscillatory circuits.

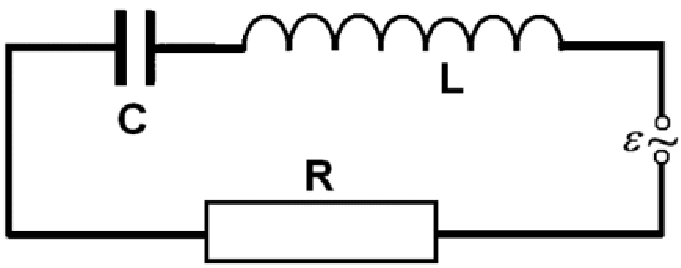

Fig. 1. RLC oscillatory circuit diagram

Based on Figure 1, we will write down Kirchhoff's second law for this circuit to consider the theory of the process occurring in the RLC oscillatory circuit under the influence of variable EMF:$e=e_o \cos\omega t$:

$$R \cdot I + U_C = e - L\frac{dI}{dt} \qquad (1)$$

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Here UC=q/C is the voltage drop across the capacitor, I=dq/dt is the current in the circuit. Let us write equation (1) as a differential equation:

$$L\frac{d^2q}{dt^2}+R\cdot\frac{dq}{dt}+\frac{q}{C}=e_o cos\omega t \qquad (2)$$

Dividing all terms of the equation by L, we obtain a differential equation with a non-homogeneous constant coefficient for the charge q:

$$\ddot{q}+\frac{R}{L}\cdot\dot{q}+\frac{q}{LC}=\frac{e_o}{L}cos\omega t \qquad (3)$$

From here we obtain a differential equation in which both damping and driving forces participate:

$$\ddot{q}+2\beta\dot{q}+\omega_o^2 q=f_o cos\omega t \qquad (4)$$

Where $\beta=\frac{R}{2L}$, $\omega_o=\sqrt{\frac{1}{LC}}$, $f_o=\frac{e_o}{L}$, As we know, mass is a measure of inertia in mechanical processes, but in electromagnetic processes the measure of inertia is inductance, and the reciprocal value of capacitance plays the role of the coefficient of elasticity in a spring pendulum.

The general solution of such a differential equation is sought as the sum of the general solution of the homogeneous equation x_1 and the particular solution of the inhomogeneous equation x_2:

$x=x_{1,general\ homogeneous}+x_{2,\ private\ heterogeneous};$

$x=x_{1,general\ homogeneous.}+x_{2,damped\ oscillation.}=A_1e^{-\beta t}\sin(\omega_1 t+\alpha_1);$

Majburlovchi kuch bo‘lmaganda, ya’ni o‘zgaruvcha EYuK bo‘lmagandagi so‘nuvchi tebranish chastotasi ifodasi quyidagicha bo‘ladi:

$$\omega_1=\sqrt{\omega_o^2-\beta^2}=\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}$$

The frequency of free oscillations of the system in the absence of a driving force, and in the case of an RLC circuit – in the absence of a variable EMF:

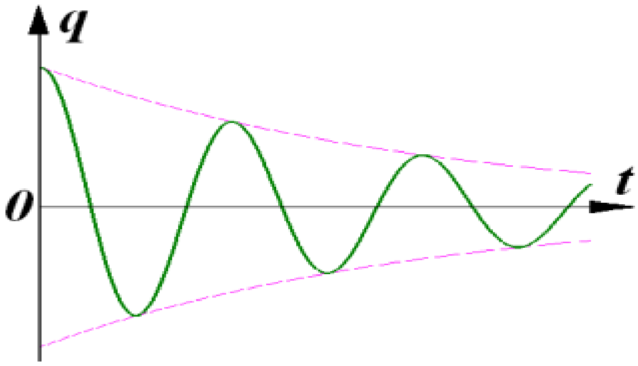

Fig. 2. Graph of free electromagnetic oscillation

The solution x_(2, partial non-uniform) defines in mechanics the nature of the motion of a system that performs steady-state forced oscillations, and for an oscillatory circuit x_(2, partial non-uniform) describes the law of change of charge on the capacitor plates under the condition of action of EMF in the oscillatory circuit $e = e_o cos\omega t$[3].

To find $x_{2,damped\ oscillation.} = A_2 cos(\omega t - \varphi_2)$ we use a vector diagram as it was done when studying mechanical vibrations:

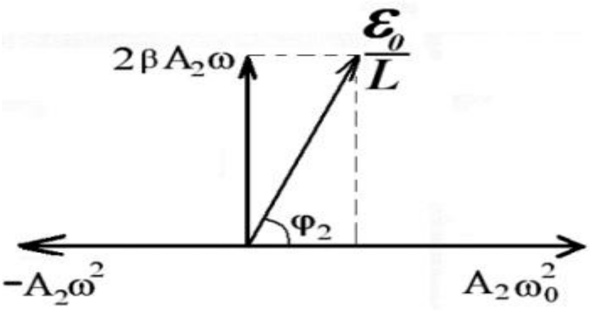

Fig.3. Vector diagram

From the vector diagram we find A_2:

$$A_2 = \frac{f_o}{\sqrt{(\omega_o^2)^2 - \omega^2 + 4\beta^2\omega^2}} = \frac{e_o/L}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + 4\left(\frac{R}{2L}\right)^2 \omega^2}}$$

Here ω is the frequency of the alternating EMF.

$\mathrm{tg}\varphi_2 = \frac{2\beta\omega}{\omega_o^2 - \omega^2} = \frac{R\omega}{L\left(\frac{1}{LC} - \omega^2\right)} = \frac{R}{\frac{1}{\omega C} - \omega L} \leftarrow sign$ $\mathrm{tg}\varphi_2$ can be negative if ω_o<ω and therefore in this case the phase shift between the driving force and the displacement $\varphi_2 > \frac{\pi}{2}$.

$$x_2 = \frac{e_o/L}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2 \omega^2}} \cos(\omega t - \varphi_2) = q(t)$$

This solution corresponds to the law of change of charge on the capacitor plates and voltage on its plates:

$$U_C = \frac{e_o/LC}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2 \omega^2}} \cos(\omega t - \varphi_2).$$

1) Let us examine the obtained solution[4].
2) If the active resistance of the circuit R tends to 0, then the attenuation coefficient of the system:
3) $\beta \to 0$ bundan $q = A_1 \sin(\omega_o t + \alpha_1) + \frac{e_o/L}{\frac{1}{LC} - \omega^2} cos\omega t$

In this case, the first term does not decay, $A_1 and\ \alpha_1$ are determined from the initial conditions. At $\beta \to 0$ $A_2(\omega \to \omega_o) \to \infty$.

4) Let's find the current in the circuit:
5) $I = \frac{dq}{dt} = -\frac{\frac{\omega e_o}{L}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\right)^2 \omega^2}} \sin(\omega t - \varphi_2).$

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The current in the circuit, and therefore the voltage on the active resistance, are shifted in phase by π/2 relative to the voltage on the capacitance and, as will be shown below, by (-π/2) relative to the voltage on the inductance[5].

Note that the phase shift of the current relative to the phase of the EMF is determined by the angle $\varphi = \varphi_2 - \frac{\pi}{2}$:

$$I = \frac{dq}{dt} = \frac{\frac{\omega e_o}{L}}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\right)^2\omega^2}}\cos\left(\omega t - \varphi_2 + \frac{\pi}{2}\right) = \frac{\frac{\omega e_o}{L}}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\right)^2\omega^2}}\cos(\omega t - \varphi)$$

At the same time tg$\varphi = \frac{1}{R}(\omega L - \frac{1}{\omega C})$.

Resonance in an oscillatory circuit. Let's find the maximum of A_2(ω) when the denominator is minimal:

$$A_2 = \frac{f_o}{\sqrt{(\omega_o^2)^2 - \omega^2 + 4\beta^2\omega^2}};$$

From this it turns out $\omega_{rez} = \sqrt{\omega_o^2 - 2\beta^2}$ ← resonant frequency - the frequency at which an EMF with amplitude eo can excite oscillations of voltage and charge on the capacitor plates with maximum amplitude. At $\beta \ll \omega_o \rightarrow R \ll \sqrt{\frac{L}{C}}$, $That's\ why\ q_{max} = q_{rez} = \frac{e_o/L}{2\beta\sqrt{-\beta^2+\omega_o^2}} \approx \frac{e_o}{R\omega_o}$.

The obtained result can be interpreted as follows: the closer the EMF frequency is to the resonant frequency of the circuit, the greater the maximum charge that occurs on the capacitor plates[6]:

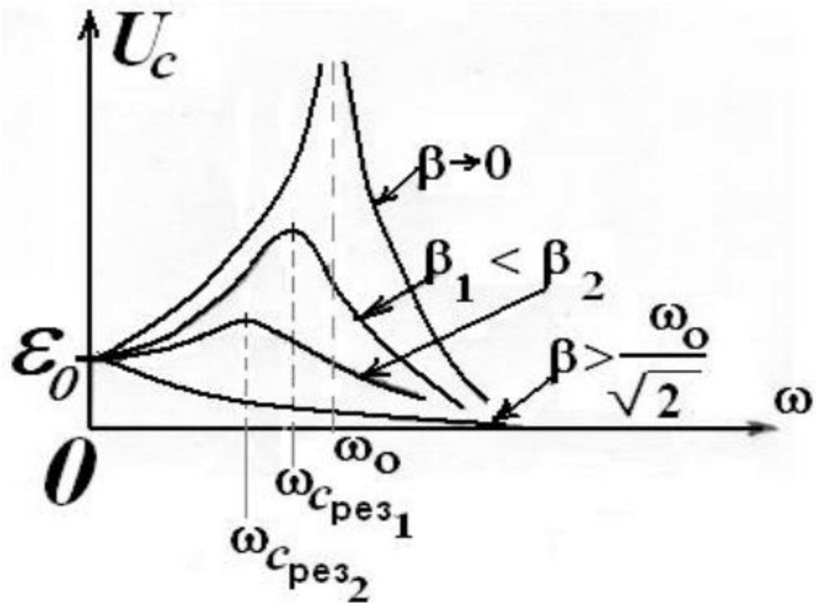

Fig.4. Graph of resonance phenomenon

From this graph it is clear that the real resonance is observed at β=0. When we compare electromagnetic resonance with mechanical ones, it is clear that in electromagnetic oscillatory processes there is a resonance of voltages on the capacitance, a resonance of current and also a resonance of voltages on the inductance, the conditions of which are the following[7]:

$$\omega_C(rez) = \sqrt{\omega_o^2 - 2\beta^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}};\ \ I_{rez} = \frac{e_o}{R};\ \ \omega_L(rez) = \frac{1}{\sqrt{LC - \frac{R^2C^2}{2}}};$$

The quality factor of the system, expressed through the resonance characteristics. Let the natural frequency ω_o≈ω_св be the frequency of free damped oscillations. For electromagnetic oscillatory processes, the quality factor is numerically equal to the ratio $Q = \frac{U_C(rez)}{e_o}$.

As a conclusion, it can be said that resonance in electromagnetic oscillatory circuits is used in the transmission and reception of electromagnetic waves. Since communication is mainly carried out through the ether using electromagnetic waves. Resonance in oscillatory circuits is also used in the military sphere as radio combat.

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