

# METHODS OF SOLVING PROBLEMS IN OLYMPIAD ISSUES IN MATHEMATICS

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## Abstract:

Today, in the continuous education system, the use of various methods and experiences is of great importance in improving the quality and efficiency of education. This article describes the importance of "Methods of solving Olympic problems by using functions".

**Keywords:** Function, concept of symmetry, quadratic equation, analytical methods of teaching, teaching methodology.

## OLIMPIADA MASALALARINI FUNKSIYAGA KELTIRIB YECHISH METODLARI

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## Annotatsiya:

Bugungi kunda uzluksiz ta'lim tizimida ta'lim sifati va samaradorligini oshirishda turli xil metod va tajribalarni qo'llash muhim ahamiyat kasb etadi. Mazkur maqolada "Olimpiada masalalarini funktsiyaga keltirib yechish metodlari" ning muhim ahamiyatga ega ekanligi bayon etilgan.

**Kalit so'zlar:** Funktsiya, simmetriklik tushunchasi, kvadrat tenglama, o'qitishning taxliliy usullari, o'qitish metodikasi.

## Abstract:

Today, the use of various methods and experiences in improving the quality and efficiency of education in the continuous education system is of great importance. This article describes the importance of "Methods of solving Olympic problems by using functions".

**Key words:** Function, concept of symmetry, quadratic equation, analytical methods of teaching, teaching methodology.



**Аннотация:** Сегодня большое значение имеет использование различных методов и опыта повышения качества и эффективности образования в системе непрерывного образования. В данной статье описывается важность «Методов решения олимпийских задач с помощью функций».

**Ключевые слова:** Функция, понятие симметрии, квадратное уравнение, аналитические методы обучения, методика обучения.

### Kirish

Hammamizga ma'lumki  $y=kx+b$  ko'rinishdagi funksiyaga to'g'ri chiziq deyiladi, bu yerda  $k$  burchak koeffitsienti  $b$  esa qandaydir son.  $y=ax^2+bx+c, (a \neq 0)$  ko'rinishdagi tenglama esa kvadrat funksiya deyiladi. Bizga ma'lumki kvadrat funksiyaning grafigi paraboladan iborat. Bizga ma'lumki agar  $a>0$  bo'lsa parabola shoxlari tepaga qaragan bo'ladi, aksincha  $a<0$  bo'lsa parabola shoxlari pasga qaragan bo'ladi. Biz grafikni chizishga to'xtalmaymiz, asosan xossalardan boshlaymiz. Oxirgi yillarda oliy ta'lim o'quv yurtlariga kirishda tushadigan imtihonlarda asosan simmetriklik tushunchasi, ya'ni qandaydir bitta funksiyani beradida boshqa chiziqqa nisbatan simmetrik funksiyasini toping yoki bo'lmasa koordinata o'qlariga nisbatan, koordinata boshiga nisbatan simmetrigini toping degan misollar tushadi, demak biz huddi shularga to'xtalamiz.

1-misol:  $f(x)=a\frac{(x-b)(x-c)}{(a-b)(a-c)}+b\frac{(x-a)(x-c)}{(b-a)(b-c)}+c\frac{(x-a)(x-b)}{(c-a)(c-b)}$  funksiya berilgan  $f(2)$  va  $f(-2)$ ni toping.

Yechish: Bu misol ham yuqoridagi misolga o'xshash misol bo'lib bunda ham  $(a-b)(a-c)(b-c) \neq 0$  shart qo'yiladi. Bu misolda agar kvadrat funksiya desak  $A>0$  bo'lishi kerak edi shuning uchun  $A$  ni ixtiyoriy tanlash uchun kvadrat uchrad  $f(x)=Ax^2+Bx+C$  ko'rinishda deb  $x$  ni o'rniqa avvalgi misolga o'xsha  $a, b$  va  $c$  larni qo'yib qiymatlarini topamiz.  $x=a$  bo'lganda  $f(a)=a, x=b$

da  $f(b)=b, x=c$  da  $f(c)=c$  hosil bo'ladi. 
$$\begin{cases} f(a) = a \\ f(b) = b \\ f(c) = c \end{cases}$$
 bundan funksiya ko'rinishi  $f(x)=x$  da

bo'larkan  $A=0, B=1$  va  $C=0$ . Endi bizdan so'ralgan narsani topadigan bo'lsak  $f(2)=2$ , hosila oladigan bo'lsak  $f(x)=1$  bundan endi xga bog'liq emas bo'lib qoladi va  $f(-2)=1$  bo'ladi. Endi bu misollarni uchinchi tipi bor uni ko'rib chiqsak.

2-misol:  $f(x)=\frac{(x-b)(x-c)}{(a-b)(a-c)}+\frac{(x-a)(x-c)}{(b-a)(b-c)}+\frac{(x-a)(x-b)}{(c-a)(c-b)}$  funksiya berilgan  $f(2)$  va  $f(-2)$ ni toping.

Yechish: Bu misolda ham  $(a-b)(a-c)(b-c) \neq 0$  shart tekshiramiz.  $x=a$  bo'lganda  $f(a)=1, x=b$

bo'lganda  $f(b)=1, x=c$  da ham  $f(c)=1$ . Endi esa 
$$\begin{cases} f(a) = 1 \\ f(b) = 1 \\ f(c) = 1 \end{cases}$$
 bundan ko'rinib turibdiki

$A=0, B=0, C=1$  bo'ladi va funksiya ko'rinishi  $f(x)=1$  bo'ladi. Bizdan so'ralgan narsani topamiz  $f(2)=1$  va hosilani olsak  $f(x)=0$  va o'rni qo'ysak  $f(-2)=0$  hosil bo'ldi. Shuning bilan ushbu ko'rinishdagi funksiyalarni barchasini kvadrat funksiya yoki to'g'ri chiziqqa keltirish mumkin va shu usulda osongina biz izlagan yechimga ega bo'lishimiz mumkin. Endi boshqa misollarni ko'rsamiz.



3-misol:  $k$  ning qanday qiymatlarida  $x^2-(k+1)x+k^2+k-32=0$  kvadrat tenglamaning bitta ildizi 2da kichik, ikkinchi ildiz 2dan katta bo`ladi?

Yechish: Kvadrat tenglama holatida ishlasak ildizlarini topish kerak va ko`plab qiyinchilik va muommolar kelib chiqadi shu sabab kvadrat funksiya deb ishlaymiz. Bu tenglamani kvadrat funksiya deb olib ya`ni kvadrat tenglamani chap tomonini olib  $f(x) = x^2-(k+1)x+k^2+k-32$  ko`rinishga keltirib olamiz va  $x_1 < 2, x_2 > 2$  bo`ladigan holatni ko`ramiz.

4-misol: Aytaylik  $P(x)$  butun koeffitsiyentli ko`phad bo`lsin va qandaydir turli butun  $a, b$  butun sonlar uchun  $P(a) \cdot P(b) = -(a-b)^2$  bo`lsin. Isbotlang:  $P(a) + P(b) = 0$ .

Isbot:  $P(a) \cdot P(b) = -(a-b)^2$

$P(a) - P(b) : (a-b)$  bo`lsa,  $u$  holda

$P(a) - P(b) = x(a-b) \quad x \in \mathbb{Z}, P(b) = y$

$y + x(a-b) = -(a-b)^2 \rightarrow y^2 + xy(a-b) + (a-b)^2 = 0$  kvadrat tenglama hosil bo`ladi.

$D = x^2(a-b)^2 - 4(a-b)^2 = (a-b)^2(x^2 - 4)$

$y$  ga bog`liq kvadrat tenglama qilsak  $D$  aniq kvadrat bo`lishi kerak bundan kelib chiqadiki  $(x^2 - 4)$  aniq kvadrat  $x = 2$ . Endi  $x$  ni o`rniga 2ni qo`yamiz

$y^2 + 2y(a-b) + (a-b)^2 = 0 \rightarrow (y+a-b)^2 = 0 \rightarrow y = b-a = P(b)$ ,  $P(a) = 2(a-b) + b - a = a-b$  va  $P(a) + P(b) = a-b + b - a = 0$  ekanligi isbotlandi.

5-misol:  $P(x) \in \mathbb{Z}(x)$  bo`lsin. Shunday  $n \in \mathbb{Z}$  borki  $P(n^2) = 0$  bo`ladi.

Isbotlang:  $\forall a \neq 0 \in \mathbb{Q}$  uchun  $P(a^2) \neq 1$ .

Isbot:  $P(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x + a_0$ ,  $a_m, a_{m-1}, \dots, a_1, a_0 \in \mathbb{Z}$

$P(n^2) = a_m (n^2)^m + a_{m-1} (n^2)^{m-1} + \dots + a_1 n^2 + a_0 = 0$ .

Faraz qilaylik,  $\exists a = \frac{p}{q}$ ,  $p \in \mathbb{Z}, q \in \mathbb{N}$ ,  $(p, q) = 1$

$(q^2)^m \cdot p(a^2) = a_m (p^2)^m + a_{m-1} (p^2)^{m-1} q^2 + \dots + a_1 p^2 (q^2)^{m-1} + a_0 (q^2)^m = (q^2)^m$

$(q^2)^m \cdot (p(a^2) - p(n^2)) = (q^2)^m = a_m ((p^2)^m - (n^2)^m) \cdot (q^2)^m + a_{m-1} ((p^2)^{m-1} - (n^2)^{m-1}) \cdot (q^2)^{m-1} + \dots + a_1 + a_0 (p^2 - n^2) \cdot (q^2)^{m-1} = (p^2 - n^2) \cdot Q(x)$

bu yerda  $Q(x) \in \mathbb{Z}(x)$ , chunki

$((p^2)^k - (n^2 \cdot q^2)^k) : p^2 - n^2 \cdot q^2 \Rightarrow (q^2)^m : p^2 - n^2 \cdot q^2 \Rightarrow (q^2)^m : (p - nq)(p + nq)$

Lekin  $\begin{cases} (q, p - nq) = (q, p) = 1 \\ (q, p + nq) = (q, p) = 1 \end{cases} \Rightarrow \pm 1 = p - nq = p + nq \Rightarrow n=0, p=\pm 1, q=1$ .

Demak, faqat  $n=0$  bo`lganda  $\exists a = \pm 1$  ekan. Bunga misol  $P(x) = x$ ,

Agar  $n \neq 0$  bo`lsa  $\Rightarrow$  ziddiyat bo`ladi.

Agar  $P(0) = 0$  bo`lsa,  $P(x) = x$  bo`lsa  $\Rightarrow \exists a = 1, P(a) = 1$ .

Agar  $n \neq 0$  bo`lsa  $\Rightarrow \nexists a \in \mathbb{Q}, P(a^2) = 1$  bo`ladigan.

6-misol: Barcha  $x \in \mathbb{R}$  uchun  $f(2011x + f(0)) = 2011x^2$  tenglikni qanoatlantiruvchi  $f: \mathbb{R} \rightarrow \mathbb{R}$  funksiyalarning barchasini toping.

Yechish:  $x \in \mathbb{R}, f(2011x + f(0)) = 2011x^2$

$x=0$  bo`lsin  $f(f(0)) = 0 \rightarrow f(0) = c \Rightarrow f(c) = 0 \Rightarrow f(2011x + c) = 2011x^2$

$x = -\frac{c}{2011} \Rightarrow f(0) = 2011 \cdot \frac{c^2}{2011^2} - \frac{c^2}{2011} = c \Rightarrow$

$c=0, c=2011$

1)  $c=0 \Rightarrow f(2011x) = 2011x^2$



$$x = \frac{y}{2011} \Rightarrow f(y) = \frac{y^2}{2011} \Rightarrow f(x) = \frac{x^2}{2011}.$$

$$(2) c=2011 \Rightarrow f(2011x+2011)=2011x^2$$

$$x = \frac{y}{2011} - 1 \Rightarrow f(y) = 2011 \left( \frac{y}{2011} - 1 \right)^2 = \frac{(x-2011)^2}{2011}. \text{ javob: } f(x) = \frac{x^2}{2011}; f(x) = \frac{(x-2011)^2}{2011}.$$

7-misol: Uchta keltirilgan kvadrat tenglamalarning diskriminantlari mos ravishda 1,4 va 9. Bu tenglamalarning har biridan bittadan ildizlarini shunday tanlash mumkinligini isbotlangki, bularning yig`indisini qolgan ildizlari yig`indisiga teng bo`ladi.

$$\text{Yechish: (1) } x^2 + ax + b = 0 \quad D = a^2 - 4c = 1$$

$$(2) x^2 + mx + n = 0 \quad D = m^2 - 4n = 4$$

$$(3) x^2 + px + q = 0 \quad D = p^2 - 4q = 9$$

$$(1) x_{1,2} = \frac{-a \pm 1}{2}; x_1 = \frac{-a-1}{2}, x_2 = \frac{-a+1}{2};$$

$$(2) x_{1,2}^I = \frac{-m \pm 2}{2}; x_1^I = \frac{-m-2}{2}, x_2^I = \frac{-m+2}{2};$$

$$(3) x_{1,2}^{II} = \frac{-p \pm 3}{2}; x_1^{II} = \frac{-p-3}{2}, x_2^{II} = \frac{-p+3}{2}.$$

$$x_1 + x_1^I + x_2^{II} = x_2 + x_2^I + x_1^{II}$$

$$\frac{-a-1}{2} + \frac{-m-2}{2} + \frac{-p+3}{2} = \frac{-a+1}{2} + \frac{-m+2}{2} + \frac{-p-3}{2}$$

$0=0$  to`g`riligi uchun isbotlandi.

8-misol:  $a, b, c, d$  haqiqiy sonlar uchun  $\begin{cases} a + b + c + d = 20 \\ ab + ac + ad + bc + bd + cd = 150 \end{cases}$  bo`lsa  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$  ni toping.

$$\text{Yechish: } \begin{cases} a + b + c + d = 20 \\ ab + ac + ad + bc + bd + cd = 150 \end{cases} \Rightarrow$$

$$\begin{cases} a^2 + b^2 + c^2 + d^2 + 2ab + 2ac + 2ad + 2bc + 2bd + 2cd = 400 \\ ab + ac + ad + bc + bd + cd = 150 \end{cases} \Rightarrow$$

$$a^2 + b^2 + c^2 + d^2 + 2 \cdot 150 = 400 \begin{cases} a^2 + b^2 + c^2 + d^2 = 100 \\ a + b + c + d = 20 \end{cases} \Rightarrow$$

$$\begin{cases} a^2 + b^2 + c^2 + d^2 = 100 \\ 10a + 10b + 10c + 10d = 200 \end{cases}$$

$$a^2 - 10a + 25 + b^2 - 10b + 25 + c^2 - 10c + 25 + d^2 - 10d + 25 = 0; (a-5)^2 + (b-5)^2 + (c-5)^2 + (d-5)^2 = 0$$

$$\Rightarrow a=b=c=d=5 \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = 0,8 \text{ Javob: } 0,8$$

9-misol:  $a, b, c$  musbat sonlar uchun  $ab+bc+ac=3$  bo`lsa, quydagi tengsizlikni isbotlang:

$$(a^7 - a^4 + 3)(b^5 - b^2 + 3)(c^4 - c + 3) \geq 27.$$

Isbot:  $a, b, c > 0$ ,  $ab+bc+ac=3$  Buni isbotlashda quydagi barcha musbat sonlarda o`rinli bo`ladigan uchta tenglikdan foydalanamiz.

$$(x^4 - 1)(x^3 - 1) \geq 0 \Leftrightarrow x^7 - x^4 + 1 \geq x^3$$

$$(x^2 - 1)(x^3 - 1) \geq 0 \Leftrightarrow x^5 - x^2 + 1 \geq x^3$$

$$(x-1)(x^3 - 1) \geq 0 \Leftrightarrow x^4 - x + 1 \geq x^3$$

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