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Using The Method of Exchange of Variables in Solving Some Quadric Equations

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Abstract:

Using The Method of Exchange of Variables in Solving Some Quadric Equations

Agar $F(f(x))=0$ (1) tenglama berilgan bo'lsa, uni $y=f(x)$ almashtirish yordamida

$F(y)=0$ (2)

ko'rinishga keltiriladi. Shundan so'ng, (2) tenglamaning hamma $y_1, y_2, \dots, y_n, \dots$ yechimlarini topib, quyidagi tenglamalar majmuasini yechishga keltiriladi.

$$f(x)=y_1,\quad f(x)=y_2,\quad \dots,\quad f(x)=y_n, \dots \qquad (3)$$

Biz (1) ko'rinishdagi to'rtinchi darajali tenglamalarning turli xususiy hollari uchun o'zgaruvchilarni almashtirish usullarini ko'rib o'tamiz.

1. $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=A$ ko'rinishdagi tenglamalarni yechish. $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=A$ (4)

bunda $\alpha, \beta, \gamma, \delta$ va A lar uchun $\alpha<\beta<\gamma<\delta$ va $\beta-\alpha=\delta-\gamma$ shartlar o'rinli bo'lsa $y=\frac{x-\alpha+x-\beta+x-\gamma+x-\delta}{4}$

almashtirish bajarib bikvadrat tenglamani yechishga keltiriladi.

1-misol. $(x+1)(x+2)(x+4)(x+5)=0$ (5)

tenglamani yeching.

Yechish. $y=\frac{x+1+x+2+x+4+x+5}{4}$

$y=x+3$ yoki $x=y-3$ almashtirish bajarib (5) tenglamani $(y-2)(y-1)(y+1)(y+2)=10$

ko'rinishda yozish mumkin.

$$(y^2-4)(y^2-1)=10 \qquad (6)$$

Bu bikvadrat tenglama 2 ta ildizga ega. $y_1=\sqrt{6}$ va $y_2=-\sqrt{6}$. Natijada (5) tenglama 2 ta ildizga ega.

$$x_1=-\sqrt{6}-3;\ x_2=\sqrt{6}-3$$

Javob: $x_1=-\sqrt{6}-3;\ x_2=\sqrt{6}-3$

2. $(ax^2+b_1x+c)(ax^2+b_2x+c)=Ax^2$ ko'rinishdagi tenglamalarni yechish.

$$(ax^2+b_1x+c)(ax^2+b_2x+c)=Ax^2 \qquad (7)$$

bunda $c\neq 0$ va $A\neq 0$, $x=0$ ildizga ega emas, shuning uchun (7) tenglamaning har 2 tomonini x^2 ga bo'lib unga teng kuchli

$$\left(ax+\frac{c}{x}+b_1\right)\left(ax+\frac{c}{x}+b_2\right)-A=0$$

tenglamani hosil qilamiz. $y=ax+\frac{c}{x}$ almashtirish bajarib kvadrat tenglamani yechishga keltiriladi.

2-misol. $(x^2+x+2)(x^2+2x+2)=2x^2$ (8)

tenglamani yeching.

Yechish. $x=0$ (8) tenglamaning ildizi bo'lmaydi, shuning uchun uni x^2 ga bo'lib unga teng kuchli

$$\left(x+1+\frac{2}{x}\right)\left(x+2+\frac{2}{x}\right)=2$$

$y=x+\frac{2}{x}$ almashtirish bajarib $(y+1)(y+2)=2$ tenglamani

tenglamani hosil qilamiz.

hosil qilamiz. Bundan $y_1 = 0$ va $y_2 = -3$. Natijada berilgan (8) tenglama quyidagi

$x + \frac{2}{x} = 0$ va $x + \frac{2}{x} = -3$ tenglamalar majmuasiga teng kuchli. Bu birlashma 2 ta $x_1 = -1$ va $x_2 = -2$ ildizga ega.

Javob: $x_1 = -1$, $x_2 = -2$.

3. $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = Ax^2$ ko'rinishdagi tenglamalarni yechish.

$$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = Ax^2 \qquad (9)$$

bunda $\alpha, \beta, \gamma, \delta$ va A lar shundayki, $\alpha\beta = \gamma\delta \neq 0$. Birinchi qavsni ikkinchisiga, uchinchi qavsni to'rtinchisiga ko'paytirib $(x^2 - x(\alpha+\beta)+\alpha\beta)(x^2 - x(\gamma+\delta)+\gamma\delta) = Ax^2$ ni

hosil

qilamiz. Endi (9) tenglamani (7) ko'rinishda yozish va yechish mumkin.

3-misol. $(x-2)(x-1)(x-8)(x-4) = 7x^2 \qquad (10)$

tenglamani yeching.

Yechish. $[(x-2)(x-4)][(x-1)(x-8)] = 7x^2$

$$(x^2 - 6x + 8)(x^2 - 9x + 8) = 7x^2$$

$x = 0$ tenglamaning ildizi bo'lmaganligi uchun uni x^2 ga bo'lamiz va unga teng kuchli tenglamani hosil qilamiz.

$$\left(x - 6 + \frac{8}{x}\right)\left(x - 9 + \frac{8}{x}\right) = 7$$

$x + \frac{8}{x} = y$ deb almashtirish bajarib $(y-6)(y-9) = 7$ kvadrat tenglamani hosil qilami

$$y_1 = \frac{15+\sqrt{37}}{2} \qquad \text{va} \qquad y_2 = \frac{15-\sqrt{37}}{2}$$

Natijada (10) tenglama quyidagi tenglamalar majmuasiga teng kuchli.

$$x + \frac{8}{x} = \frac{15+\sqrt{37}}{2}; \qquad x + \frac{8}{x} = \frac{15-\sqrt{37}}{2}$$

Bu birlamaning birinchi tenglamasi yechimi

$$x_1 = \frac{\frac{15+\sqrt{37}}{2} \quad \sqrt{\left(\frac{15+\sqrt{37}}{2}\right)^2 - 32}}{2}, \qquad x_2 = \frac{\frac{15+\sqrt{37}}{2} \quad \sqrt{\left(\frac{15+\sqrt{37}}{2}\right)^2 - 32}}{2},$$

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Birlashmaning 2-tenglamasi yechimga ega emas. Demak, berilgan tenglama 2 ta ildizga ega

Javob: $x_1 = \dfrac{15+\sqrt{37}+\sqrt{30\sqrt{37}+134}}{4}$, $x_2 = \dfrac{15+\sqrt{37}-\sqrt{30\sqrt{37}+134}}{4}$.

4. $a(cx^2+p_1x+q)^2+b(cx^2+p_2x+q)^2 = Ax^2$ ko'rinishdagi tenglamalarni yechish.

$$a(cx^2+p_1x+q)^2+b(cx^2+p_2x+q)^2 = Ax^2 \qquad (11)$$

bunda a, b, c, q, A sonlar, $q \neq 0$, $A \neq 0$, $c \neq 0$, $b \neq 0$.

$x = 0$ ildizga ega emas. Shuning uchun (11) tenglamani x^2 ga bo'lib unga teng kuchli

$$a\left(cx+\frac{q}{x}+p_1\right)^2+b\left(cx+\frac{q}{x}+p_2\right)^2 = A$$

tenglamani hosil qilamiz. $y = cx+\dfrac{q}{x}$ almashtirish bajarib kvadrat tenglamani yechishga keltiriladi.

4-misol. $3(x^2+2x-1)^2-2(x^2+3x-1)^2+5x^2 = 0 \qquad (12)$

tenglamani yeching.

Yechish. $x = 0$ (12) tenglamaning ildizi bo'lmaganligi uchun har ikki tomonini x^2 ga bo'lib,

$$3\left(x+2-\frac{1}{x}\right)^2-2\left(x+3-\frac{1}{x}\right)^2+5 = 0 \qquad (13)$$

(12) tenglamaga teng kuchli tenglamani hosil qilamiz. $x-\dfrac{1}{x} = y$ almashtirish bajarib (13) tenglamani

$$3(y+2)^2-2(y+3)^2+5 = 0 \qquad (14)$$

ko'rinishda yozamiz. (14) kvadrat tenglama 2 ta $y_1 = 1$, $y_2 = -1$ ildizga ega.

Shuning uchun (13) tenglama quyidagi tenglamalar majmuasiga teng kuchli.

$$x-\frac{1}{x} = 1 \quad \text{va} \quad x-\frac{1}{x} = -1 \qquad (15)$$

(15) tenglamalar majmuasi to'rtta ildizga ega.

$$x_1 = \frac{-1-\sqrt{5}}{2},\quad x_2 = \frac{-1+\sqrt{5}}{2},\quad x_3 = \frac{1-\sqrt{5}}{2},\quad x_4 = \frac{1+\sqrt{5}}{2}$$

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