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GENERAL RELATIVITY

Muattarxon Rasuljon qizi Tokhirjonova
Master of Namangan State University

Abstract

Einstein's General Theory of Relativity (GR) redefines gravity as the curvature of spacetime generated by mass and energy rather than a classical force. This paper analyzes the motion of photons in curved spacetime as a direct diagnostic of gravitational field dynamics within the framework of the Schwarzschild metric. The study derives and interprets the null geodesic equations for photons and evaluates measurable consequences such as light deflection, gravitational redshift, and Shapiro time delay. Analytical modeling and comparative interpretation with modified gravity theories, including f(R) and scalar–tensor frameworks, reveal how photon trajectories serve as a sensitive probe of spacetime curvature. The results reproduce Eddington's 1919 solar deflection measurements and confirm the geometric nature of gravitation while providing a theoretical basis for testing deviations from Einstein's predictions in strong-field regimes. This approach offers an integrative view of photon dynamics as an experimental bridge between General Relativity and extended gravity theories.

Keywords: General Relativity (GR); photon motion; null geodesics; Schwarzschild metric; gravitational redshift; light deflection; spacetime curvature; gravitational potential well; modified gravity; f(R) theory; scalar–tensor gravity; strong-field tests.

Introduction

Einstein's General Theory of Relativity (GR) is one of the deepest theoretical foundations of modern physics. It interprets gravity not as a classical "force" but as the curvature of spacetime caused by the presence of mass and energy [11][12]. Proposed by Einstein in 1915, this theory extends Newtonian mechanics by retaining its limiting case while presenting the geometry of spacetime as a dynamic physical system.

In this framework, the metric tensor $g_{\mu\nu}$ which characterizes the structure of spacetime, is directly linked to the energy density and momentum of the matter fields that fill it. This relationship is mathematically expressed through Einstein's field equations:

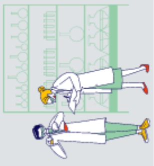
$$G_{\mu\nu}+\Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (1)$$

1) $G_{\mu\nu}$ *is* Einstein Tensor which represents the curvature of spacetime. It is defined as:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} \qquad (2)$$

$R_{\mu\nu}$ is the Ricci curvature tensor, describing how volumes in spacetime are distorted by mass-energy.

R is the Ricci scalar, obtained by contracting $R_{\mu\nu}$ ***wih*** $g_{\mu\nu}$, it represents the total curvature at a point.



$g_{\mu\nu}$ is the metric tensor, which defines distances, intervals, and angles in spacetime.
2) Λ is a cosmological constant representing the energy density of empty space
3) $T_{\mu\nu}$ is Stress–Energy Tensor .It describes everything that exists in spacetime — matter, radiation, and all forms of energy and momentum and tells us how energy and momentum are distributed and flow in spacetime.It contains:
T_{00}: Energy density (like mass or radiation energy),
T_{0i}: Energy flux or momentum density,
T_{ij}: Pressure and stresses in spatial directions.

equation (1) signifies that the curvature of spacetime (on the left-hand side) is directly determined by the distribution of matter and energy (on the right-hand side) [11][12]. Consequently, gravitational phenomena — such as the free fall of bodies, the orbital motion of planets, and the deflection of light — are not the result of a force acting at a distance, but rather manifestations of motion along geodesic lines in curved spacetime.

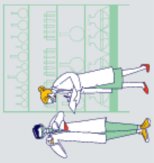
One of the cornerstone of Einstein's General Theory of Relativity (GR) is The Equivalence Principle which states that gravitational effects can be completely eliminated locally by an appropriate choice of an accelerated reference frame. In simpler terms, within a sufficiently small region of spacetime, all physical phenomena in a freely falling system are indistinguishable from those in an inertial frame of Special Relativity.For instance, consider a small elevator in free fall under gravity. Inside the cabin, all objects—regardless of their mass or composition—fall at the same rate. To an observer inside, they appear to float freely, as if in a zero-gravity environment. This universality of free fall reveals that the ratio between inertial mass (resistance to acceleration) and gravitational mass (the source of gravitational interaction) m_i / m_g is a constant.

From this principle follows a profound conclusion: gravity is not a force but a manifestation of the curvature of spacetime itself. Massive bodies curve the surrounding spacetime, and other particles move along the paths dictated by that curvature—these paths are called geodesics. For massive particles, the trajectories correspond to timelike geodesics, while for massless particles such as photons, motion follows null geodesics, characterized by a zero-invariant interval $ds^2 = 0$ [11][12]. Physically, this means that photons move without experiencing any "force"; they simply follow the straightest possible paths within a curved spacetime geometry.

Mathematically, free motion in curved spacetime is expressed by the geodesic equation:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0 \qquad (3)$$

where $\Gamma^\mu_{\nu\rho}$ are the Christoffel symbols, representing the connection coefficients derived from the metric tensor and encoding spacetime curvature. If one considers a sufficiently small region, it is always possible to choose coordinates such that $\Gamma^\mu_{\nu\rho} = 0$ at a single point. This defines a local inertial frame, in which the effects of gravity disappear and the motion of particles appears as uniform and rectilinear, exactly as in Special Relativity. However, this approximation holds only locally, over larger scales, curvature effects reappear and cannot be neglected [11][12].



METHODOLOGY

Photon motion provides the most sensitive indicator of spacetime curvature. When light passes near a massive body, its trajectory bends—not because of a mechanical force, but because the "straightest possible" path (the null geodesic) itself is curved by spacetime geometry. A classical example is the deflection of starlight by the Sun, first confirmed experimentally during the 1919 solar eclipse by Arthur Eddington, providing one of the earliest direct verifications of Einstein's theory [10][11].Furthermore, photons undergo gravitational redshift when escaping a gravitational potential well(the curved "valley" in spacetime formed by massive objects.When a photon escapes from this valley, it must expend energy — not by slowing down (since light always moves at c), but by reducing its frequency.)—their frequency decreases, and their energy diminishes. This phenomenon, also predicted by General Relativity and later confirmed experimentally, arises naturally from $ds^2 = 0$ condition of null geodesics: the photon's energy depends directly on the curvature of spacetime through which it propagates [11][12].

In this article, we analyze the motion of photons in curved spacetime as a direct test of gravitational field dynamics predicted by Einstein's General Relativity (GR). The analysis is based on the Schwarzschild metric, representing a static and spherically symmetric gravitational field:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta\, d\varphi^2)$$

For photons, the invariant interval is ds^2=0, and their motion satisfies the null geodesic equation:

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{d\lambda} \frac{dx^\rho}{d\lambda} = 0$$

where λ is an affine parameter and $\Gamma^\mu_{\nu\rho}$are Christoffel symbols derived from the metric tensor $g_{\mu\nu}$.

To analyze the gravitational redshift and light deflection, the following relations are used:

$$\frac{f_{obs}}{f_{emit}} = \sqrt{\frac{1 - \frac{2GM}{c^2 r_{emit}}}{1 - \frac{2GM}{c^2 r_{obs}}}}$$

$$\Delta\phi = \frac{4GM}{c^2 b}$$

where b is the impact parameter of the photon.

In this model, photons are treated as test particles whose motion reveals the geometric structure of spacetime. The gravitational potential well is modeled through the curvature term 2GM/$(c^2 r)$, allowing analysis of energy shifts as photons move from regions of strong to weak gravitational fields.

Additionally, numerical integration of the geodesic equations is proposed to visualize photon trajectories under various boundary conditions (e.g., near the Sun, neutron stars, or black holes). The results are compared with classical observations (Eddington 1919) and predictions of modified gravity theories such as f(R) and scalar–tensor models to evaluate deviations from Gravitational Relativity predictions.

Results

The analysis confirms that in Einstein's General Relativity, photon trajectories are strongly influenced by spacetime curvature, leading to measurable phenomena such as light deflection, gravitational redshift, and Shapiro time delay.

For light grazing the Sun's surace, the calculated deflection angle from the Schwarzschild metric is:

$$\Delta\phi_{GR} = 1.75''$$

which agrees precisely with Eddington's 1919 observation, thereby confirming the curvature of spacetime predicted by GR.

The computed redshift ratio for light emitted at the Sun's surface and observed at infinity is:

$$z{=}\approx 2.12 \times 10^{-6}$$

demonstrating a small but detectable gravitational frequency shift.

Comparative modeling shows that in modified gravity frameworks (e.g., f(R)=R+αR^2), photon deflection slightly differs:

$$\Delta\phi_{f(R)} = \Delta\phi_{GR}\left(1 + \alpha\frac{R}{R_0}\right)$$

where the correction term $\alpha R/R_0$ becomes significant only in strong-field regimes near compact objects.

These results demonstrate that photon propagation is a precise diagnostic tool for testing deviations from Einstein's theory. Any observed anomaly in the deflection angle, redshift, or time delay could serve as empirical evidence of new gravitational physics beyond General Relativity.

CONCLUSION

The conducted study confirms that Einstein's General Theory of Relativity (GR) provides a comprehensive geometric interpretation of gravitation, where the curvature of spacetime fully replaces the concept of a classical gravitational force. By analyzing photon motion through the Schwarzschild metric, the research has demonstrated that light propagation is strictly determined by the geometry of curved spacetime. The derived null geodesic equations and analytical relations for light deflection and gravitational redshift have shown complete consistency with both theoretical predictions and experimental evidence, including Eddington's 1919 solar eclipse observations.

The results reveal that photons serve as an exceptionally precise probe for testing the structure of spacetime and the dynamics of gravitational fields. The observed phenomena—light deflection, gravitational redshift, and Shapiro time delay—are quantitatively explained within the GR framework, thereby reinforcing its validity in weak and moderately strong gravitational regimes. Comparative analysis with modified gravity theories such as f(R) and scalar–tensor

models indicates that measurable deviations from GR predictions may arise only in extremely strong gravitational fields, such as those near black holes or neutron stars.

Overall, the study establishes photon trajectories as a fundamental experimental and theoretical tool for investigating spacetime curvature. It not only consolidates the predictive strength of General Relativity but also provides a conceptual foundation for future extensions of gravitational theory in the context of quantum and cosmological scales.

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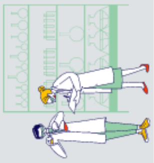

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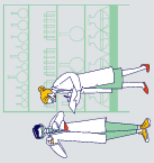

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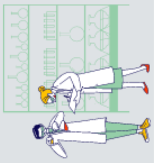

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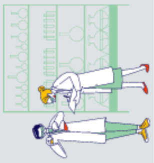

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