SOME APPLICATIONS IN ECONOMICS

Egamov J. A.
Sobitov R. A.
NamSU

Abstract
Today for those who receive education in economic specialties (including second education), and meets the requirements of state educational standards for economic disciplines. The material is presented practically without proofs - the main emphasis is on acquiring skills in using the mathematical apparatus. Each issue is accompanied by numerous characteristic solutions to problems and relevant economic applications. Translated with DeepL.com (free version)

Keywords: Undefined and definite integrals, economics.

Introduction

The study of mathematical disciplines and their economic applications, which form the basis of actual economic mathematics, will allow the future specialist not only to acquire the necessary basic skills used in economics, but also to form the components of his thinking: level, outlook and culture. All this will be necessary for successful work and for orientation in future professional activity.

Basic formula of integral calculus

THEOREM 1. A continuous function f(x) on the segment [a, b] has a prime on this segment. One of the primes is the function

$$F(x) = \int_a^x f(t)\,dt. \tag{1}$$

In formula (1), the variable of integration is denoted by the letter t to avoid confusion with the upper variable limit of x.

Since every other prime differs from F(x) by a constant value, the relationship between the indefinite and definite integrals is as follows

$$\int f(x)\,dx = \int_a^x f(t)\,dt + C,$$

where C is an arbitrary constant.

According to Theorem 1.4, a continuous function f(x) on the segment [a, b] has a prime, which is defined by the formula

$$\int_a^x f(t)\,dt = F(x) + C, \tag{2}$$

where C is some constant. Substituting in (2) x = a, taking into account property 1 of the definite integral, we obtain

$$\int_a^a f(t)\,dt = F(a) + C, \quad 0 = F(a) + C, \quad \text{откуда } C = -F(a).$$

Then from (2) we have

$$\int_a^x f(t)\,dt = F(x) - F(a).$$

Assuming x = b, we obtain the formula

$$\int_a^b f(x)\,dx = F(b) - F(a). \tag{3}$$

Equality (3) is called the basic formula of integral calculus, or the Newton-Leibniz formula. The difference F(b) - F(a) is conventionally written by the symbol F(x) , i.e., F(x).

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b. \tag{4}$$

Formula (4) gives a wide range of possibilities for calculating indefinite integrals. It is necessary to calculate the indefinite integral and then find the difference of the first integral according to (4).

Some applications in economics

Generally speaking, in economic problems variables change discretely. To use the definite integral, it is necessary to make some idealized model that assumes continuous change of dependent variables (functions) and independent variables (argument). Let us consider the corresponding examples.

Daily output

Find the daily output P for a working day of eight hours, if labor productivity during the day varies according to the empirical formula

$$p = f(t) = p_0(-0{,}2t^2/t_0^2 + 1{,}6t/t_0 + 3),$$

where t - time in hours, p0 - dimension of productivity (output per hour), t0 - dimension of time (h). This formula quite reflects the real process of work (Fig. 1): productivity first grows, reaching a maximum in the middle of the working day at t = 4 h, and then falls.

Web of Discoveries: Journal of Analysis and Inventions
webofjournals.com/index.php/3

Web of Discoveries: Journal of Analysis and Inventions
webofjournals.com/index.php/3

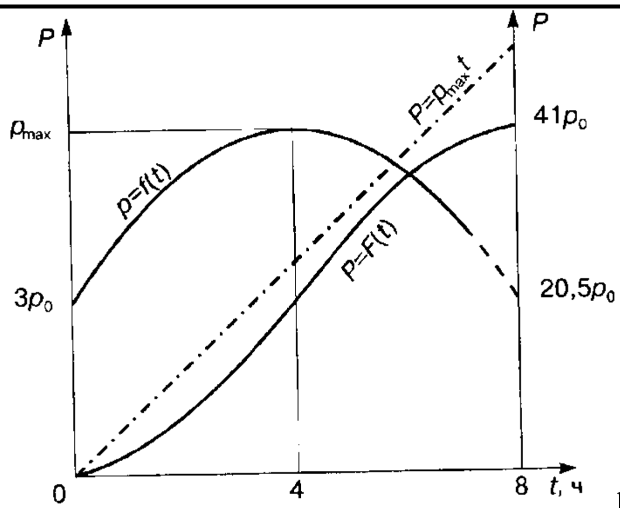

рис 1

SOLUTION. Assuming that productivity varies continuously during the day, i.e. p is a continuous function of the argument t on the segment

[0, 8], the daily output P can be expressed by a certain integral:

$$P = p_0 \int_0^8 (-0{,}2t^2/t_0^2 + 1{,}6t/t_0 + 3)dt = F(t)\Big|_0^8 =$$

$$= p_0(-0{,}2\frac{t^3}{3}/t_0^2 + 0{,}8t^2/t_0 + 3t)\Big|_0^8 =$$

$$= 41{,}06 p_0 t_0 = 41{,}06 a_0,$$

where a0 is a multiplier having the dimension of a unit of production. If during the whole day the work was carried out rhythmically and with maximum productivity Rtah = 6.2p0, then the daily output would be Pmah = 49.6a0, or about 21% more. Fig. 7.7 illustrates the solution of the problem: daily output is numerically equal to the area of the curvilinear trapezoid bounded from above by the curve f(t); the second curve shows the growth of output in time (the graph of the first-order F(t) corresponds to the right axis of ordinates P). The value T = 4 h corresponds to the inflection point of the F(t) curve: in the first half of the working day the intensity of output is higher than in the second half. The dashed line P = pmaht corresponds to the output with uniform productivity pmah.

Equipment output at constant growth rate

Production of equipment of a certain type is characterized by the growth rate of its output

$$K = \frac{\Delta y}{\Delta t}\frac{1}{y},$$

where Δy is the increase in the output of this equipment for the time interval Δt, and y is the level of its production per unit of time at time t. Find the total amount of equipment produced at time t, assuming that K is a known constant value, the unit of time is a year, and at the initial moment of time t = 0 the level of annual production of equipment was y0.

SOLUTION. Let us pass to the limit at Δt → 0, assuming that it exists. We will also assume that y is a continuous function of time t. According to the definition of the production function

$$K = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} \frac{1}{y} = \frac{y'}{y} = (\ln y)'.$$

Integrating this equality in the range from 0 to t, we obtain

$$\ln y \Big|_{y_0}^{y} = Kt \Big|_0^t = Kt, \text{ или } \ln \frac{y}{y_0} = Kt,$$

$$\text{откуда } y = y_0 e^{Kt}.$$

The total amount of equipment produced during the time period t is given by a certain integral

$$Y(t) = \int_0^t y(t)\,dt = \int_0^t y_0 e^{Kt} dt =$$

$$= \frac{1}{K} y_0 e^{Kt} \Big|_0^t = \frac{1}{K} y_0 (e^{Kt} - 1).$$

For example, at K = 0.05 (5% annual growth rate), the total amount of equipment produced over 10 years will be as follows

$$Y(10) = 20 y_0 (e^{0,5} - 1) \approx 13 y_0,$$

with production levels increasing by almost 65% over this time period.

References

1. Красс М.С., Чупрынов Б.П. Основы математики и ее приложения в экономическом образовании. Учеб. — 2-е изд., испр. — М.: Дело, 2001.
2. В.А. Бабайцев, А.В. Браилов, А.С. Солодовников. Математика в экономике. Часть 5. Руководство к решению задач. Теория вероятностей. М., Финансовая академия при Правительстве РФ, 1999.
3. М.И. Баканов, А.Д. Шеремет. Теория экономического анализа. — М., "Финансы и статистика", 1994.
4. Н.П. Башарин. Начала финансовой математики. - М., "ИНФРА-М", 1995.
5. В.Е. Гмурман. Теория вероятностей и математическая статистика. — М., "Высшая школа", 1998.

Web of Discoveries: Journal of Analysis and Inventions
webofjournals.com/index.php/3