Volume 2, Issue 11, November 2024 **ISSN (E):** 2938-3757

EXAMPLES OF INCREASE AND DECREASE OF FUNCTION

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Abstract
The article examines examples of increasing and decreasing functions.

Keywords: increasing, decreasing, function.

Introduction

Let us consider a function $y = f(x)$defined on an interval of a numerical axis $(a, b) \subset R = (-\infty, +\infty)$and introduce the following definitions.

Definition 1. The function $y = f(x)$is called *non-decreasing* on the interval (a, b), if for any values $x_1, x_2$from this interval $x_1, x_2 \in (a, b)$satisfying the inequality $x_1 < x_2$the inequality is satisfied

$$f(x_1) \le f(x_2).$$

Definition 2. The function $y = f(x)$is called *non-increasing* on the interval (a, b)if for any values $x_1, x_2$from this interval $x_1, x_2 \in (a, b)$satisfying the inequality $x_1 < x_2$the inequality is satisfied

$$f(x_1) \ge f(x_2).$$

Definition 3 . The function $y = f(x)$is called *increasing* on the interval (a, b), if for any values $x_1, x_2$from this interval $x_1, x_2 \in (a, b)$satisfying the inequality $x_1 < x_2$the inequality is satisfied

$$f(x_1) < f(x_2).$$

Definition 4. The function $y = f(x)$is called *decreasing* on the interval (a, b), if for any values $x_1, x_2$from this interval $x_1, x_2 \in (a, b)$satisfying the inequality $x_1 < x_2$the inequality is satisfied

$$f(x_1) > f(x_2).$$

According to definitions 1–2, a non-decreasing function on an interval $(a, b)$is characterized by the fact that with an increase in the value of the argument, the corresponding value of the function does not decrease, and an increasing function on an interval $(a, b)$is characterized by the fact that with an increase in the value of the argument, the corresponding value of the function does not increase. Similarly, according to definitions 3–4, an increasing function on an interval $(a, b)$is characterized by the fact that with an increase in the value of the argument, the corresponding value of the function increases, and a decreasing (a, b)function on an interval is characterized by the fact that with an increase in the value of the argument, the corresponding value of the function decreases.

Example 1. Consider a function $f(x) = x^2$on the interval $(-\infty, 0)$.

For any values satisfying $x_1, x_2 \in (-\infty, 0)$the inequality $x_1 < x_2$we have the inequality

$$f(x_1) - f(x_2) = x_1^2 - x_2^2 = (x_1 - x_2)(x_1 + x_2) > 0, \text{ i.e.}$$

$$f(x_1) > f(x_2).$$

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This means that by definition 4 the given function is decreasing on the interval $(-\infty, 0)$.

Example 2. Let's consider the function

$$f(x) = \begin{cases} -x^2 - 2x + 3, & -5 < x \le -1 \\ 4, & -1 < x \le 1 \\ \dfrac{15 - 7x}{2}, & 1 \le x < 3 \\ x^2 - 6x + 6, & 3 < x < 6 \end{cases}$$

on the interval $(-5,6)$.

The graph of this function looks like

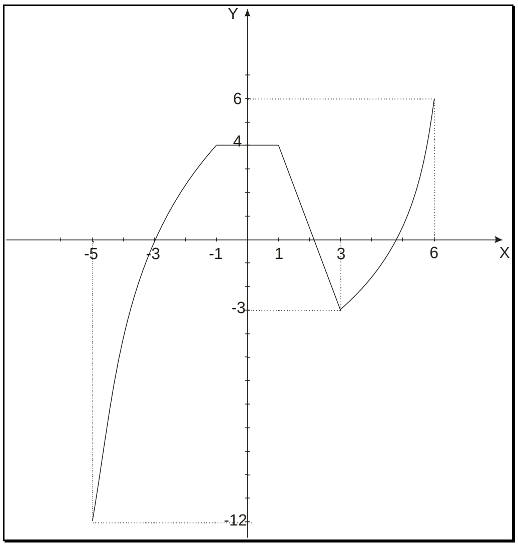


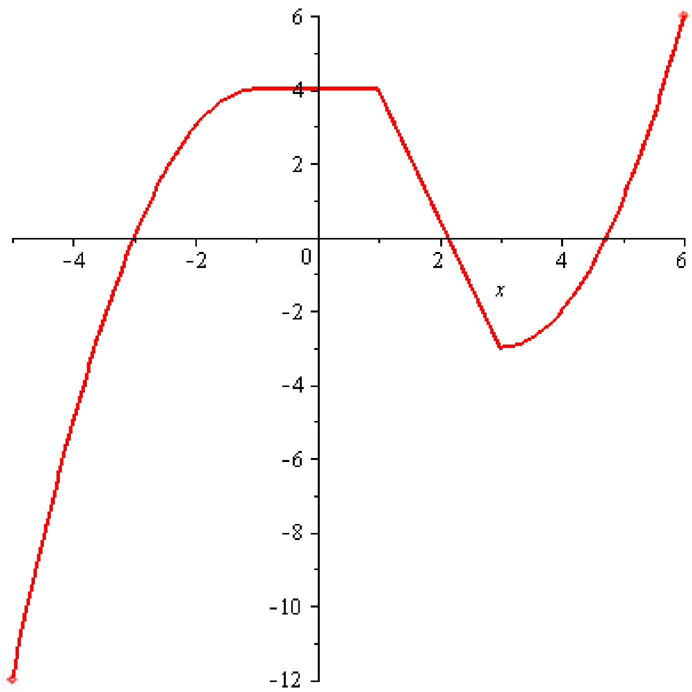
Fig. 1.

In the Maple program it looks like this.

> $h := x \rightarrow piecewise\left(-5 \le x \,\mathbf{and}\, x \le -1, -x^2 - 2\cdot x + 3, -1 < x \,\mathbf{and}\, x \le 1, 4, 1 < x \,\mathbf{and}\, x < 3, \dfrac{(15 - 7\cdot x)}{2}, 3 \le x \,\mathbf{and}\, x \le 6, x^2 - 6\cdot x + 6\right)$

$$h := x \rightarrow piecewise\left(-5 \le x \text{ and } x \le -1,\ -x^2 - 2x + 3,\ -1 < x \text{ and } x \le 1, 4, 1 < x \text{ and } x < 3, \frac{15}{2} - \frac{7}{2}x, 3 \le x \text{ and } x \le 6, x^2 - 6x + 6\right)$$

> $plot(h(x), x = -5\,..6, color = red, thickness = 2, discont = true)$



The function under consideration on intervals $(-5,-1)$and $(3,6)$is increasing, decreasing on the interval, $(-5,1)$non-decreasing on the interval, $(-1,3)$non-increasing on the interval, and $(-1,1)$is both non-increasing and non-decreasing on the interval.
If a function is increasing on some interval, then it is also non-decreasing on that interval. Similarly, if a function is decreasing on an interval, then it is not increasing on it.
The converse statements do not always hold, i.e. not every non-decreasing function on an interval is increasing on it, and not every non-increasing function is decreasing on it. The function considered in example 2 is non-decreasing $f(x)$on an interval $(-1,1)$, but is not increasing on this interval. Similarly, on the interval $(-1,1)$the function $f(x)$is not increasing, but is not decreasing.
Definition 5. **A** variable is called *monotonic* if it changes in only one direction, i.e. either only increases or only decreases.
The movement of point *x* towards the positive direction of the abscissa axis is monotonically increasing, and in the opposite direction it is monotonically decreasing.
Definition 6. *Any* increasing or decreasing function on an arbitrary interval is called *monotone. function* on it.
Definition 7. Intervals on which a function is increasing or decreasing are called *intervals of monotonicity* of the function.
To determine the intervals of monotonicity and the intervals on which the function is non-increasing or non-decreasing, one can use the following theorems using the differentiability of the function.
Theorem 1. Let the function $y = f(x)$be differentiable on the interval (a, b). For this function to be non-decreasing on the interval (a, b)it is necessary and sufficient that the relation be satisfied $y' = f'(x) \geq 0$on this interval.

Theorem 2. Let the function $y = f(x)$be differentiable on the interval (a, b). For this function not to increase on the interval (a, b)it is necessary and sufficient that the relation is satisfied $y' = f'(x) \leq 0$on this interval.
Theorem 3. Let the function $y = f(x)$be differentiable on the interval $(a, b)$and $f'(x) > 0$for all $x \in (a, b)$. Then this function is increasing on this interval.
Theorem 4. Let the function $y = f(x)$be differentiable on the interval $(a, b)$and $f'(x) < 0$for all $x \in (a, b)$. Then this function is decreasing on this interval.
Example 3. Let us consider, over a five-hour time interval, $(0,5)$the law expressing the speed of a car that makes a rectilinear uniformly variable motion in one direction, as a function of time:

$$v(t) = v_0 + at, t \in (0,5).$$

Here $v_0$is the initial velocity $(v_0 > 0)$, $a$is the acceleration, $t$is the time. When $a > 0$the motion is uniformly accelerated, and when it $a < 0$is uniformly decelerated. We will find the intervals of increase and decrease of the function $v(t)$, when $t \in (0,5)$, using Theorems 3–4. To do this, we will find its derivative $v'(t), t \in (0,5)$:

$$v'(t) = a, t \in (0,5).$$

Let $a > 0$. Then $v'(t) > 0$when $t \in (0,5)$, i.e. the function $v(t)$is increasing on the interval $(0,5)$according to Theorem 3.

Let $a < 0$. Then the car will stop at the moment in time at $t_1$which the speed is zero, i.e. $v(t) = 0$. This moment in time is equal to $t_1 = \frac{v_0}{a}$. If $t_1 > 5$, then on the interval under consideration the function $v(t)$is decreasing. If , $t_1 < 5$then on the interval $(0, t_1)$the function $v(t)$ является is decreasing, and on the interval $(t_1, 5)$it is constant. (the car does not move (assuming that the car moves only in one direction)).

Example 4. Find the intervals of monotonicity of the function

$$f(x) = 3x - x^3.$$

Solution : Let's find the derivative of this function:

$$f'(x) = 3 - 3x^2.$$

Having resolved the inequality $f'(x) < 0$, i.e.

$$3(1 - x^2) < 0$$

we get that it is valid for all values $x$of the set $(-\infty, -1) \cup (1, +\infty)$. For all values $x$from the interval $(-1,1)$the inequality is satisfied $f'(x) > 0$, i.e.

$$3(1 - x^2) > 0.$$

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This means, according to Theorems 3–4, on the interval $(-1,1)$the function $f(x)$is increasing, and on intervals $(-\infty,-1)$, $(1,+\infty)$is decreasing.

Thus, the intervals of monotonicity of a given function are intervals $(-\infty,-1),(-1,1),(1,+\infty)$.

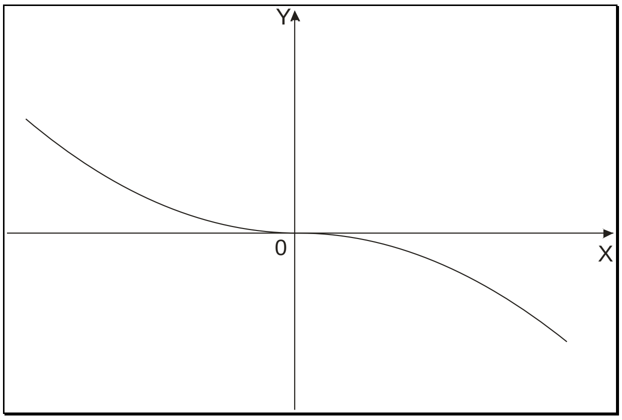

Fig. 2.

In the Maple program, a function graph.

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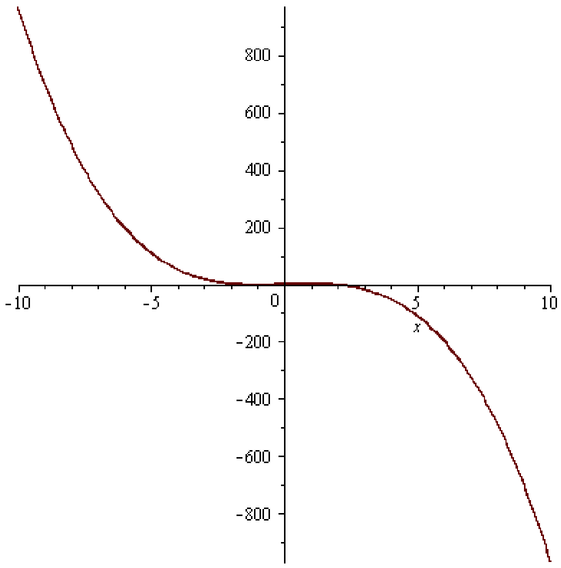
\>$plot(3\cdot x - x^3)$;



Example 5. Find the intervals of monotonicity of the function

$$f(x) = \frac{e^x}{x}.$$

Solution . The domain of definition of this function is the entire real axis, except for the point $x = 0$at which the denominator vanishes, i.e. $R\backslash\{0\} = (-\infty, 0) \cup (0, +\infty)$. The derivative function is equal to

$$f'(x) = \frac{e^x(x-1)}{x^2}.$$

To determine the intervals of increase of a given function, we solve the inequality $f'(x) > 0$, i.e.

$$\frac{e^x(x-1)}{x^2} > 0$$

in the domain of the function $f(x)$. The inequality is equivalent to the inequality

$$(x-1) > 0.$$

The set of solutions of the last inequality is an infinite interval $(1, +\infty)$.

The set of solutions of the inequality $f'(x) < 0$, i.e.

$$\frac{e^x(x-1)}{x^2} < 0$$

The domain of a function $f(x)$is the intervals $(-\infty, 0), (0,1)$.

This means that, according to Theorems 3–4, this function increases on the interval $(1, +\infty)$, and decreases on the intervals $(-\infty, 0), (0,1)$.

Thus, the intervals of monotonicity of a given function are intervals $(-\infty, 0), (0,1), (1, +\infty)$.

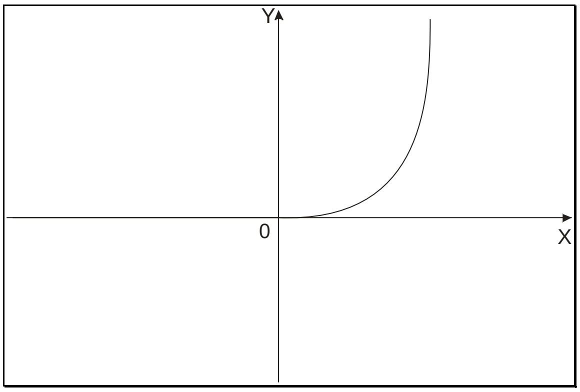
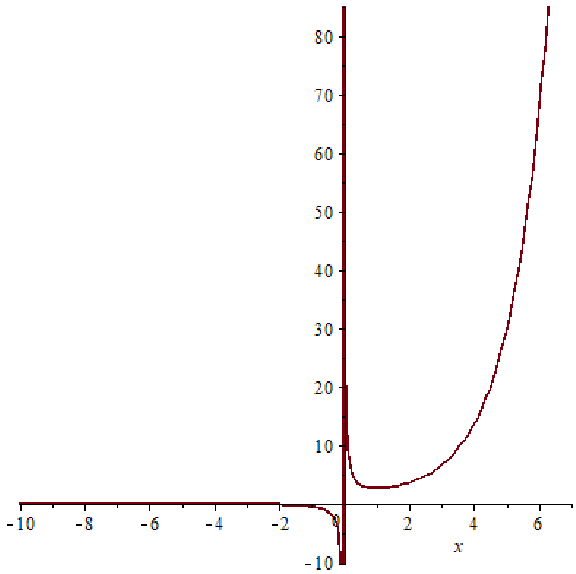


Fig. 3.

In maple we use the plot (exp (x)/ x) command ;

PlotBuilder (e^x /x);



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