ANALYSIS OF THE FREQUENCY CHARACTERISTICS OF LONGITUDINAL AND RADIAL VIBRATIONS IN A CIRCULAR CYLINDRICAL SHELL

Yalgashev B. F.
E-mail: byalgashev@bk.ru

Abstract
The paper addresses the problem of harmonic longitudinal-radial vibrations in a circular cylindrical shell with free ends. The solution is based on refined oscillation equations for the shell, which were derived earlier from an exact three-dimensional formulation and its solution through transformations. A thorough review of studies on harmonic and nonstationary processes in elastic bodies is presented, focusing on both classical theories (such as Kirchhoff-Love and Flyugge) and refined Timoshenko-type theories (like Hermann-Mirsky and Filippov-Khudoinazarov). Four frequency equations are derived for the main components of the longitudinal and radial displacements of the shell, with special cases corresponding to thin-walled shells. Using the solutions to these frequency equations, the natural vibration frequencies of the shell, including for thin-walled versions, are determined. A comparative frequency analysis of the longitudinal vibrations of a circular cylindrical elastic shell is conducted based on the classical Kirchhoff-Love theory and the refined Hermann-Mirsky and Filippov-Khudoinazarov theories. The results lead to conclusions about the applicability of the studied oscillation equations, depending on the shell's waveform and length. Specifically, it is concluded that all the equations are unsuitable for describing wave processes in short shells, where the lengths are comparable to their transverse dimensions.

Keywords: Frequency equation, waveforms, shear deformation, inertia of rotation.

Introduction

In many areas of science and technology, particularly in physics and mechanics, researchers aim to simplify the analysis of wave behavior by reducing it to the study of basic harmonic waves [1]. However, the reverse process translating the characteristics of a harmonic wave back into an analysis of general wave motion in a material with specific initial conditions presents significant challenges [2]. Despite these difficulties, considerable attention is given to studying harmonic processes in elastic bodies. This focus arises because even at an intermediate stage of problem-solving, valuable information can be obtained about key properties of oscillatory systems, such as phase and group velocities, natural frequencies, and vibration modes [3]. These studies are typically based on refined Timoshenko-type equations, which account for transverse shear deformation and rotational inertia [4]. In developing new theories of shell vibrations, efforts are made to derive refined vibration equations that incorporate specific physical, mechanical, or geometric factors [5].

The methods for deriving oscillation equations, based on the dynamic theory of elasticity, are categorized into several approaches depending on the factors considered. Monographs provide a comprehensive analysis of scientific studies focused on deriving vibration equations and advancing refined theories of deformable solids, particularly circular cylindrical layers, shells, and rods, along with an in-depth examination of various aspects of this problem. In works [6,7], general equations for the longitudinal and transverse vibrations of viscoelastic plates, rods, and cylindrical shells were derived using the three-dimensional formulation of problems in the linear theory of viscoelasticity. These equations also account for environmental factors and friction forces. The anisotropic properties and temperature effects on the plates and rods were considered in the related theory. From the exact equations, approximate forms similar to those of S.P. Timoshenko and others were derived, which include higher-order derivatives with respect to both coordinates and time. Using these exact and refined approximate equations, specific vibration problems for rods, plates, and shells are solved.

In monograph [9], this method was extended to a circular cylindrical layer interacting with a deformable solid medium and an ideal liquid, considering the viscoelastic properties of the layer material and various contact modes between the layers and the medium. For the first time, an intermediate surface was introduced as the primary surface that conveys information about the layer's oscillations. This surface can transition, in limiting cases, to the inner, outer, or middle surface of the layer, depending on the values of a certain parameter χ, which has a continuous spectrum bounded from both above and below.

It is important to note that in works [5,8], a method was developed for deriving vibration equations based on the application of general solutions in transformations of three-dimensional elasticity problems. This method relies on integral transformations in both coordinates and time, along with the use of general solutions in the transformation of three-dimensional elasticity problems. These solutions are then expanded in power series to approximately satisfy the dynamic conditions specified on the boundary surfaces of the elastic system under consideration [10, 11]. A significant and successful application of this method to dynamic problems was achieved in works [12-13]. In these studies, the general equations of vibration for circular cylindrical shells and rods were derived, accounting for the interaction with a viscous fluid and the rotation of the rod. The core of the method involves examining the constructed solutions under various types of external influences to determine the conditions under which displacements, or their "main components," satisfy simple oscillation equations. Additionally, an algorithm was developed that enables the calculation of approximate field values, such as displacement and stress, in any section at any given moment in time based on the field of these “main component” displacements. In the works of authors [14,15], oscillation equations for circular cylindrical viscoelastic shells and layers interacting with a liquid were developed. These developments were made without relying on additional physical or mechanical hypotheses or assumptions, allowing for the derivation of both classical and refined oscillation equations. An algorithm was proposed that enables the precise determination of the stress-strain state at any point in an arbitrary section of the system, based on the values of the desired functions using the field of these functions.

Currently, the analysis of vibrations in elements of engineering structures, such as rods, plates, and shells, is conducted using both classical (Kirchhoff-Love) and refined (Timoshenko-type) theories. In most of these studies, there is a strong emphasis on considering factors such as rotational inertia, transverse shear deformation, and the multilayer structure of the materials [16,17]. Additionally, considerable attention is given to incorporating the rheological, particularly the viscoelastic, properties of the material [18,19], as well as the interaction of structures with deformable media, such as viscous fluids [20] or dispersive waves [21]. The study of natural frequencies and modes of vibration of rods, plates, and shells remains highly relevant. This is evident from publications addressing the impact of boundary condition deviations on frequency characteristics [22], as well as those discussing biharmonic analysis [23] and frequency analysis [24,25].

This article focuses on a circular cylindrical elastic shell, with the goal of studying its harmonic longitudinal-radial oscillations using both classical and refined theories. The objective is to conduct a comparative analysis of the numerical values of the natural longitudinal-radial oscillation frequencies of the elastic cylindrical shell. These values are obtained using the equations proposed by the authors, as well as those based on the classical Kirchhoff-Love theory, the refined theories of Hermann-Mirsky (of the S.P. Timoshenko type), and Khudoinazarov Kh. The refined oscillation equations developed by the authors in [14, 15] are used as one of the equations in the refined theories.

II. Methods

2.1. Theoretical framework.

In a cylindrical coordinate system (r,θ,z), we examine the natural longitudinal-radial vibrations of a circular cylindrical elastic shell, which is freely supported at both ends. The shell has a length l and inner and outer radii, denoted by r_1, and r_2, respectively. The directions of the coordinate axes, as well as the radii and displacement directions, are illustrated in Figure 1. It is assumed that the shell is not subjected to any external forces, and both of its surfaces are free from external loads.

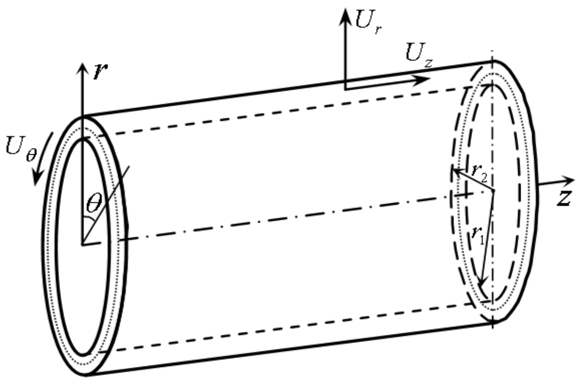


Fig.1. Geometry of the shell

In reference [14], the general equations of oscillation for a circular cylindrical shell were derived, and these equations were subsequently extended in [15] to account for the interaction between the shell and a viscous fluid. For the purpose of solving the problem, we assume that the terms related to the fluid interaction are neglected, and that the shell's surfaces are free from external loads. With these assumptions, we then transform the equations into dimensionless form using the following formulas.

$$U_{r,0}=U^*_{r,0};\;\; U_{r,1}=r_1U^*_{r,1};\;\; U_{z,0}=r_1U^*_{z,0};$$

$$U_{z,1}=U^*_{z,1};\;\; z=r_1z^*;\;\; r=r_1r^*;\;\; t=\frac{r_1}{b}t^*.$$

For convenience in notation, we omit the asterisks above the values in the following expressions, resulting in the simplified form

$$q_1U_{r,0}+2\nu q_1\frac{\partial U_{z,0}}{\partial z}-\left(2-\frac{1}{2}\partial_2\right)U_{r,1}+\frac{1}{2}\frac{\partial U_{z,1}}{\partial z}=0,$$

$$(2\nu q_1)\frac{\partial U_{r,0}}{\partial z}+\left[\frac{q_1}{q_2}\partial_1\right]U_{z,0}-2\frac{\partial U_{r,1}}{\partial z}-2U_{z,1}=0,$$

$$q_1U_{r,0}+2\nu q_1\frac{\partial U_{z,0}}{\partial z}+\left[\left(q_2\ln\frac{r_2}{r_1}-\frac{1}{2}\right)\partial_2+2\frac{r_1^2}{r_2^2}\right]U_{r,1}-\left[\frac{q_2}{q_1}\ln\frac{r_2}{r_1}+\frac{1}{2}\right]\frac{\partial U_{z,1}}{\partial z}=0, \quad (1)$$

$$2\nu q_1\frac{\partial U_{r,0}}{\partial z}+\frac{q_1}{q_2}\partial_1U_{z,0}-\left[2\nu q_2\partial_2\ln\frac{r_2}{r_1}+2\frac{r_1^2}{r_2^2}\right]\frac{\partial U_{r,1}}{\partial z}-\left[\left(2q_2\frac{\partial^2}{\partial z^2}+\lambda_2\right)\ln\frac{r_2}{r_1}+2\frac{r_1^2}{r_2^2}\right]U_{z,1}=0,$$

where

$$\partial_1=\frac{b^2}{a^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2};\quad \partial_2=\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial z^2};\; q_1=-\frac{1}{1-2\nu};\quad q_2=-\frac{1}{2(1-\nu)};$$

ν - represents the Poisson's ratio of the material of the layer; a - the speed of propagation of longitudinal waves in the shell material is denoted by the appropriate term. The boundary conditions for the problem of natural vibrations of a cylindrical shell, with its ends freely supported at $z=0$ and $z=l$, where l is the length of the shell, will take the form given in [9].

$$U_{z,0}=\frac{\partial^2U_{z,0}}{\partial z^2}=0;\;\; U_{z,1}=\frac{\partial^2U_{z,1}}{\partial z^2}=0;\;\; \frac{\partial U_{r,0}}{\partial z}=\frac{\partial^3U_{r,0}}{\partial z^3}=0;\;\; \frac{\partial U_{r,1}}{\partial z}=\frac{\partial^3U_{r,1}}{\partial z^3}=0. \quad (2)$$

Note that the conditions (2) are derived from the displacement values, which are determined using the formulas given by

$$U_r(r,z,t)=\frac{r}{2}U_{r,0}(z,t)-\frac{1}{2}U_{r,1}(z,t),\;\; U_z(r,z,t)=U_{z,0}-\frac{r}{4}U_{z,1}. \quad (3)$$

Thus, the problem of natural longitudinal-radial vibrations of an elastic circular cylindrical shell is reduced to solving the equations in (1) subject to the boundary conditions in (2), which are satisfied term by term in the series.

$$U_{r,0}=\sum_m^\infty W_{0,m}(t)\cos(\gamma_m z) \qquad U_{r,1}=\sum_m^\infty W_{1,m}(t)\cos(\gamma_m z)$$

$$U_{z,0}=\sum_m^\infty U_{0,m}(t)\sin(\gamma_m z) \qquad U_{z,1}=\sum_m^\infty U_{1,m}(t)\sin(\gamma_m z)$$

Volume 2, Issue 12, December 2024 ISSN (E): 2938-3757

where $\gamma_m = \dfrac{m\pi}{l}$, l-the length of the shell $m = 0,1,2,3,....$. Substituting these series into the system of equations (1), we obtain

$$q_1 W_{0,m}(t) + 2\nu q_{12}\gamma_m U_{0,m}(t) - \left(2 - \frac{1}{2}\bar{\partial}_2\right) W_{1,m} - \frac{1}{2}\gamma_m U_{1,m}(t) = 0,$$

$$-2\nu q_1 \gamma_m W_{0,m}(t) + \frac{q_1}{q_2}\bar{\partial}_1 U_{0,m}(t) + 2\gamma_m W_{1,m}(t) - 2U_{1,m}(t) = 0,$$

$$q_1 W_{0,m}(t) + 2\nu q_1 \gamma_m U_{0,m}(t) + \left[\left(q_2 \ln\frac{r_2}{r_1} - \frac{1}{2}\right)\bar{\partial}_2 + 2\frac{r_1^2}{r_2^2}\right] W_{1,m}(t) - \left(\frac{q_2}{q_1}\ln\frac{r_2}{r_1} + \frac{1}{2}\right)\gamma_m U_{1,m}(t) = 0,$$

$$-2\nu q_1 \gamma_m W_{0,m}(t) + \frac{q_1}{q_2}\bar{\partial}_1 U_{0,m}(t) - \left[2\nu q_2 \bar{\partial}_2 \ln\frac{r_2}{r_1} + 2\frac{r_1^2}{r_2^2}\right]\gamma_m W_{1,m}(t) + \tag{4}$$

$$+\left[\left(2q_2\gamma_m^2 + \bar{\partial}_2\right)\ln\frac{r_2}{r_1} - 2\frac{r_1^2}{r_2^2}\right] U_{1,m}(t) = 0,$$

where $\bar{\partial}_1 = (1 - q_1)\dfrac{\partial^2}{\partial t^2} + \gamma_m^2$, $\bar{\partial}_2 = \dfrac{\partial^2}{\partial t^2} + \gamma_m^2$.

For system (4) to have a nonzero solution, it is necessary that its main determinant, formed by the coefficients of the unknown functions $U_{i,m}(t)$ and $W_{i,m}(t)$, $(i = 1,2)$ be equal to zero. Let this determinant be denoted by Δ_1, and introduce the following notation:

$$\omega_1 = \left(q_2 \ln\frac{r_2}{r_1} - \frac{1}{2}\right)\bar{\partial}_2 + 2\frac{r_1^2}{r_2^2}; \quad \omega_2 = \frac{q_2}{q_1}\ln\frac{r_2}{r_1} + \frac{1}{2};$$

$$\omega_3 = 2\nu q_2 \bar{\partial}_2 \ln\frac{r_2}{r_1} + 2\frac{r_1^2}{r_2^2}; \quad \omega_4 = \left(2q_2\gamma_m^2 - \bar{\partial}_2\right)\ln\frac{r_2}{r_1} - 2\frac{r_1^2}{r_2^2}. \tag{5}$$

$$\Delta_1 = \begin{vmatrix} q_1 & 2\nu q_1 \gamma_m & -\left(2 - \frac{1}{2}\bar{\partial}_2\right) & -\frac{1}{2}\gamma_m \\ -\gamma_m 2\nu q_1 & \frac{q_1}{q_2}\bar{\partial}_1 & 2\gamma_m & -2 \\ q_1 & 2\nu q_1 \gamma_m & \omega_1 & \omega_2 \gamma_m \\ -2\nu q_1 \gamma_m & \frac{q_1}{q_2}\bar{\partial}_1 & \omega_3 \gamma_m & \omega_4 \end{vmatrix}$$

Expanding this determinant by the elements of the third row, we obtain

$$\Delta_1 = q_1 A_{31} + 2\nu q_1 \gamma_m A_{32} + \omega_1 A_{33} + \omega_2 \gamma_m A_{34}$$

where $A_{3i} = (-1)^{3+i} D_{3i}$ is an algebraic complement, D_{3i} are minors of the third row elements a_{3i}.

The resulting expression for the determinant can be rewritten as

$$\Delta_1 = q_1 D_{31} - 2\nu q_1 \gamma_m D_{32} + \omega_1 D_{33} - \omega_2 \gamma_m D_{34}.$$

Putting instead of ω_i $\left(i = \overline{1,4}\right)$ their expressions (5), we finally get

$$\Delta_1 = a_1\bar{\partial}_1\bar{\partial}_2^2 - \left(a_2\bar{\partial}_1 - a_4\bar{\partial}_2\right)\bar{\partial}_2 + a_3\bar{\partial}_1 - a_5\bar{\partial}_2 + a_6 \quad (6)$$

where

$$a_1 = \frac{q_1^2}{q_2}\ln\left(\frac{r_2}{r_1}\right)\left(\frac{3}{2} - q_2\ln\frac{r_2}{r_1}\right); \quad a_4 = -4\nu^2 q_1^2 \ln\left(\frac{r_2}{r_1}\right)\left(q_2\ln\frac{r_2}{r_1} - 1\right);$$

$$a_2 = \frac{q_1^2}{q_2}\left[2q_2\gamma_m^2(q_2+\nu)\ln^2\frac{r_2}{r_1} - \left(\nu q_1\gamma_m^2 + \nu q_2\gamma_m + 2q_2\frac{r_1^2}{r_2^2} + 2q_2 + 2\right)\ln\frac{r_2}{r_1} - \frac{1}{2}\nu\gamma_m(2\nu\gamma_m - \gamma_m - 1)\right];$$

$$a_3 = \frac{q_1^2}{q_2}\left[4q_2\gamma_m^2\left(1 + \frac{1}{2q_1} - \frac{1}{2q_1}\frac{r_1^2}{r_2^2}\right)\ln\frac{r_2}{r_1} + 2\gamma_m^2\left(1 - \frac{r_1^2}{r_2^2}\right) - 4\nu\gamma_m^2 + 2\nu\gamma_m\left(2 - \frac{r_1^2}{r_2^2}\right) + 4\left(1 - 2\frac{r_1^2}{r_2^2}\right)\right];$$

$$a_5 = 2\nu^2 q_1^2\left[2\nu q_2\gamma_m^2\ln\frac{r_2}{r_1} + 2q_2\gamma_m^2\ln\frac{r_2}{r_1} - 2\frac{r_1^2}{r_2^2} + 2 + -4\ln\frac{r_2}{r_1} - \right.$$

$$\left. -4\left(q_2\ln\frac{r_2}{r_1} - \frac{1}{2}\right)\left(\gamma_m + q_2\gamma_m^2\ln\frac{r_2}{r_1} - \frac{r_1^2}{r_2^2}\right) + 4\gamma_m\frac{r_1^2}{r_2^2}\ln\frac{r_2}{r_1} + 4\nu q_2\gamma_m^3\left(\frac{q_2}{q_1}\ln\frac{r_2}{r_1} + \frac{1}{2}\right)\ln\frac{r_2}{r_1}\right];$$

$$a_6 = 2\nu^2 q_1^2\left[2\gamma_m^2\left(1 - \frac{r_1^2}{r_2^2}\right) + 8\left(1 + \frac{r_1^2}{r_2^2}\right) - 8q_2\gamma_m^2\ln\frac{r_2}{r_1} + 8\gamma_m^2\frac{r_1^2}{r_2^2}\left(\gamma_m + q_2\gamma_m^2\ln\frac{r_2}{r_1} - \frac{r_1^2}{r_2^2}\right) - \right.$$

$$\left. -2\left(2\frac{r_1^2}{r_2^2} - \gamma_m\right)\gamma_m^3\left(\frac{q_2}{q_1}\ln\frac{r_2}{r_1} + \frac{1}{2}\right)\right].$$

Putting into (6), the values of the differential operators $\bar{\partial}_1$ and $\bar{\partial}_2$ by formulas (4), we finally get

$$\Delta_1 = a_1(1-q_1)\frac{\partial^6}{\partial t^6} + \left[a_1\gamma_m^2(3-2q_1) - a_2(1-q_1) - a_4\right]\frac{\partial^4}{\partial t^4} +$$

$$+\left[a_1\gamma_m^4(3-q_1) - \gamma_m^2(a_2(2-q_1) + 2a_4) + a_3(1-q_1)\right]\frac{\partial^2}{\partial t^2} + a_1\gamma_m^6 - (a_2+a_4)\gamma_m^4 + (a_3-a_5)\gamma_m^2.$$

(7)

Hence, each of the functions $U_{0,m}(t)$, $U_{1,m}(t)$, $W_{0,m}(t)$, and $W_{1,m}(t)$ must satisfy the equation

$$\Delta_1\zeta_m(t) = 0, \quad (8)$$

where $\zeta_m(t)-$ is any of the above functions. Then, based on (3), the displacements U_r and U_z must satisfy the same equation.

2.2. Frequency equations.

In equation (8), we put $\zeta_m(t) = A_m e^{\omega t}$ and get the following frequency equation

$$a_1(1-q_1)\omega^6 + \left[a_1\gamma_m^2(3-2q_1) - a_2(1-q_1) - a_4\right]\omega^4 +$$
$$+\left[a_1\gamma_m^4(3-q_1) - \gamma_m^2(a_2(2-q_1) + 2a_4) + a_3(1-q_2)\right]\omega^2 +$$
$$+ a_1\gamma_m^6 - (a_2+a_4)\gamma_m^4 + (a_3-a_5)\gamma_m^2 + a_6 = 0 \quad (9)$$

The derived equation is general for both the layer and the shell. To obtain a simplified frequency equation for the shell, it is enough to set $\ln(r_2/r_1)=0$ in the expressions for the coefficients a_i. In this case, the coefficients $a_i\,(i=\overline{1,6})$ take the following form:

$$a_1=0;\;\; a_2=-\frac{q_1^2}{2q_2}\nu\gamma_m(2\nu\gamma_m-\gamma_m-1);\;\; a_3=\frac{2q_1^2}{q_2}\left[\gamma_m^2(1-\frac{l_1^2}{2_2^2})-2\nu\gamma_m^2+\nu\gamma_m(2-\frac{r_1^2}{r_2^2})+2(1-2\frac{r_1^2}{r_2^2})\right];$$

$$a_4=0;\;\; a_5=2(1+\gamma_m-\frac{r_1^2}{r_2^2});\;\; a_6=\gamma_m^4+6\frac{r_1^2}{r_2^2}\gamma_m^3+2(1-\frac{r_1^2}{r_2^2}-\frac{r_1^4}{r_2^4})\gamma_m^2+8(1+\frac{r_1^2}{r_2^2}).$$

The equation in (9) takes the following form:

$$a_2(1-q_1)\omega^4+\left[a_2(2-q_1)\gamma_m^2-a_3(1-q_1)\right]\omega^2+(a_2+a_4)\gamma_m^4+(a_5-a_3)\gamma_m^2+a_6=0\,. \qquad (10)$$

To conduct a comparative analysis, we will calculate the frequencies of natural longitudinal-radial oscillations of an elastic circular cylindrical shell using the oscillation equations from different theories. These theories include the classical Kirchhoff-Love theory [6], as well as the refined theories of Herman-Mirsky [4] and Fillipov-Khudoinazarov [5]. The corresponding frequency equations are as follows:

Fillipov-Khudoinazarov

$$\omega^6+\left[(g_2+g_3)\gamma_m^2+g_5\omega_1\right]\omega^4+\left[(g_4+g_2g_3)\gamma_m^4+(1+g_2)g_5\omega_1\gamma_m^2+\omega_1\right]\omega^2+ \\ +g_2g_4\gamma_m^6+g_2g_5\omega_1\gamma_m^4+g_2g_6\omega_1^2\gamma_m^2=0, \qquad (11)$$

Herman-Mirsky

$$\omega^6+\frac{2y_1}{1-\nu}\left[k_T+y_1y_4+y_1(g+k)\right]\gamma_m^4+\frac{4y_1}{(1-\nu)^2}\left[y_1(1+2k_T)\gamma_m^4+2\left(y_1y_4+\frac{k_T-k_E^2}{2}\right)\gamma_m^2- \\ -y_2\gamma_m^2+y_4k_T\right]\omega^2+y_1k_T\gamma_m^6+(k_T+y_1y_4-y_2)\gamma_m^6+(y_4k_T-y_3)\gamma_m^2=0, \qquad (12)$$

Kirchhoff-Love

$$\omega^4+\frac{2}{1-\nu_0}\left(\frac{1}{3}\gamma_m^4+\gamma_m^2+\frac{1}{\xi^2}\right)\omega^2+\left[\frac{4}{3(1-\nu)^2}\gamma_m^6+\frac{4(1+\nu)\cdot\gamma_m^2}{(1-\nu)\xi^2}\right]=0, \qquad (13)$$

where

$$g_1=\frac{3-4\nu}{8(1-2\nu)(1-\nu^2)};\quad g_2=\frac{3(1-\nu)(1+4\nu)}{3-4\nu};\quad g_3=3-2\nu;\quad g_4=2-2\nu;\quad g_5=3+2\nu;$$

$$g_6=2+2\nu;\quad y_1=\frac{1}{3}\left(1-\frac{1}{\xi^2}\right);\quad y_2=\frac{3k_T\nu}{3\xi^2}+k_T^2+\frac{\nu^2}{3\xi^2};\quad y_3=k_T\frac{\nu^2}{\xi^2};\quad k_T=\frac{1-\nu}{2}k^2;$$

$$y_4=\frac{1}{\xi^2}\left(1+\frac{1}{3\xi^2}\right);\quad \xi=\frac{R}{h};$$

R - radius of the middle surface, h - thickness of the shell , k_T - the correction coefficient of the Tymoshenko.

Web of Technology: Multidimensional Research Journal
webofjournals.com/index.php/4

Web of Technology: Multidimensional Research Journal
webofjournals.com/index.php/4

III. Results and Discussions

The frequency equations (9) - (13) were solved numerically using the MAPLE application software, with the following geometric data for the shell: $r_1 = 1{,}0; r_2 = 1{,}1; h = 0{,}1$. Poisson's ratio was taken to be $\nu = 0{,}2$. The Timoshenko coefficient was set to 5/6. Table 1 presents the numerical values of the shell frequencies, which are dependent on the waveform values calculated using equations (9) and (10). In Fig. 2, the dependence of the frequency ω on the waveform γ_m is plotted based on the obtained numerical values. Table 2 presents the numerical values of the shell frequencies as a function of the waveform values, calculated using equations (11) - (13). In Fig. 3, the frequency ω is plotted against the waveform γ_m according to both the classical and refined equations of various theories, based on the obtained numerical values. From Tables 1 and 2, it can be observed that the real parts of the roots of the frequency equations, except for those derived from the Hermann-Mirsky equations, are negative. According to the physical interpretation of the problem and based on the Hurwitz criterion [26], it follows that the roots of the frequency equations (which are cubic and quadratic equations with respect to ω^2) must be purely imaginary.

The numerical values of the roots show that, in reality, such results are applicable to all equations, except for the Hermann-Mirsky equation (Table 2), up to a certain value of the

Table 1

γ_m		Equation (9)			Equation (10)	
		ω_1	ω_2	ω_3	ω_1	ω_2
0,1	Д М	0 ±0,1624	0 ±2,3557	0 ±19,9508	0 ±1,0539	0 ±0,1549
0,3	Д М	0 ±0,4837	0 ±2,3659	0 ±19,9511	0 ±1,0539	0 ±0,4647
0,5	Д М	0 ±0,8062	0 ±2,3859	0 ±19,9517	0 ±1,0539	0 ±0,7745
0,7	Д М	0 ±1,1287	0 ±2,4154	0 ±19,9527	0 ±1,0844	0 ±1,0539
0,9	Д М	0 ±1,4512	0 ±2,4542	0 ±19,9539	0 ±1,3942	0 ±1,0539
1,1	Д М	0 ±1,7736	0 ±2,5019	0 ±19,9554	0 ±1,7041	0 ±1,0539
1,3	Д М	0 ±2,0961	0 ±2,5578	0 ±19,9572	0 ±2,0139	0 ±1,0539
1,5	Д М	0 ±2,4186	0 ±2,6214	0 ±19,9594	0 ±2,3237	0 ±1,0539
1,7	Д М	0 ±2,6921	0 ±2,7411	0 ±19,9619	0 ±2,6336	0 ±1,0539
1,9	Д М	0 ±2,7694	0 ±3,0636	0 ±19,9647	0 ±2,9434	0 ±1,0539
2,1	Д М	0 ±2,8526	0 ±3,3861	0 ±19,9679	0 ±3,2533	0 ±1,0539
2,3	Д М	0 ±2,9412	0 ±3,7086	0 ±19,9714	0 ±3,5631	0 ±1,0539
2,5	Д М	0 ±3,0347	0 ±4,0311	0 ±19,9753	0 ±3,8729	0 ±1,0539

parameter γ_m and at particular values of the shell's transverse dimensions. In contrast, the Hermann-Mirsky equations produce such results only for specific values of the Timoshenko coefficient $k_T > 1$, , which is inherently not possible.

IV. Conclusions

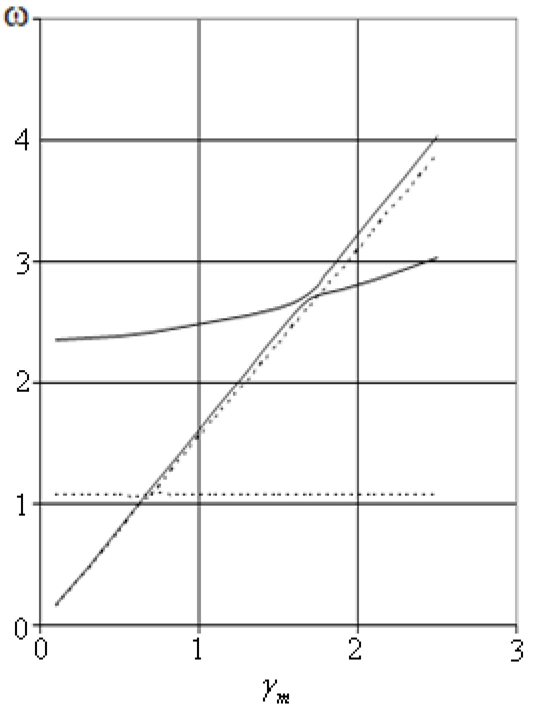
A comparison of the numerical results obtained from equations (9) and (10) shows that equation (9) yields six frequency values, while the equation for the shell provides four frequencies. Transitioning from equation (9) to equation (10) for the shell results in the loss of two frequencies. The obtained numerical results, at $r_1 = 1{,}0; r_2 = 1{,}1; h = 0{,}1; \nu = 0{,}2;\ k_T = \frac{5}{6}$, are shown in Table 2 and presented as curves in Fig. 3. From Table 2 and Fig. 3, the following conclusions can be drawn:

- The Hermann-Mirsky equation does not satisfy the Hurwitz criterion and provides inaccurate results. More precise results, which align with the Hurwitz criterion, can only be obtained at specific values of the Timoshenko correction coefficient k_T. This conclusion is entirely consistent with the corresponding conclusion in work [9].

Table 2

		Equation (9)			**Equation Kirchhoff-Love**		**Equation Hermann-Mirsky**			**Equation Filippov-Khudoynazarov**		
γ_m		ω_1	ω_2	ω_3	ω_1	ω_2	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0,1	Д М	0 ±0,1624	0 ±2,3557	0 ±19,9508	0 ±0,018839	0 ±0,169107	0 ±0,040546	0 ±0,242211	0 ±0,971814	0 ±0,2569	0 ±1,9919	0 ±3,2114
0,3	Д М	0 ±0,4837	0 ±2,3659	0 ±19,9511	0 ±0,089294	0 ±0,507118	0 ±0,047928	±0,041815 ±0,741225	∓ 0,041815 ±0,741225	0 ±0,7707	0 ±2,0070	0 ±3,2319
0,5	Д М	0 ±0,8062	0 ±2,3859	0 ±19,9517	0 ±0,244502	0 ±0,845170	0 ±0,071230	±0,394764 ±0,891635	∓ 0,394764 ±0,891635	0 ±1,2845	0 ±2,0370	0 ±3,2723
0,7	Д М	0 ±1,1287	0 ±2,4154	0 ±19,9527	0 ±0,478457	0 ±1,183228	0 ±0,126816	±0,574947 ±1,050513	∓ 0,574947 ±1,050513	0 ±1,7983	0 ±2,0817	0 ±3,3316
0,9	Д М	0 ±1,4512	0 ±2,4542	0 ±19,9539	0 ±0,790637	0 ±1,521289	0 ±0,197236	±0,730863 ±1,217192	∓ 0,730863 ±1,217192	0 ±2,1408	0 ±2,3121	0 ±3,4086
1,1	Д М	0 ±1,7736	0 ±2,5019	0 ±19,9554	0 ±1,180941	0 ±1,859350	0 ±0,273994	±0,878904 ±1,390692	∓ 0,878904 ±1,390692	0 ±2,2136	0 ±2,8259	0 ±3,5016
1,3	Д М	0 ±2,0961	0 ±2,5578	0 ±19,9572	0 ±1,649339	0 ±2,197414	0 ±0,353344	±1,024269 ±1,569865	∓ 1,024269 ±1,569865	0 ±2,2994	0 ±3,3397	0 ±3,6091
1,5	Д М	0 ±2,4186	0 ±2,6214	0 ±19,9594	0 ±2,195816	0 ±2,535483	0 ±0,433502	±1,168888 ±1,753639	∓ 1,168888 ±1,753639	0 ±2,3974	0 ±3,7296	0 ±3,8535
1,7	Д М	0 ±2,6921	0 ±2,7411	0 ±19,9619	0 2,820277	0 2,873646	0 ±0,513624	±1,313506 ±1,941105	∓ 1,313506 ±1,941105	0 ±2,5064	0 ±3,8614	0 ±4,3673
1,9	Д М	0 ±2,7694	0 ±3,0636	0 ±19,9647	0 3,211564	0 3,523057	0 ±0,593333	±1,458395 ±2,131531	∓ 1,458395 ±2,131531	0 ±2,6256	0 ±4,0034	0 ±4,8811
2,1	Д М	0 ±2,8526	0 ±3,3861	0 ±19,9679	0 3,549640	0 4,303758	0 ±0,672487	±1,603637 ±2,324337	∓ 1,603637 ±2,324337	0 ±2,7539	0 ±4,1541	0 ±5,3949
2,3	Д М	0 ±2,9412	0 ±3,7086	0 ±19,9714	0 3,887705	0 5,162542	0 ±0,751056	±1.749235 ±2,519070	∓ 1.749235 ±2,519070	0 ±2,8903	0 ±4,3127	0 ±5,9088
2,5	Д М	0 ±3,0347	0 ±4,0311	0 ±19,9753	0 4,225768	0 6,099400	0 ±0,829065	±1,895161 ±2,715377	∓ 1,895161 ±2,715377	0 ±3,0338	0 ±4,4780	0 ±6,4226

Web of Technology: Multidimensional Research Journal
webofjournals.com/index.php/4



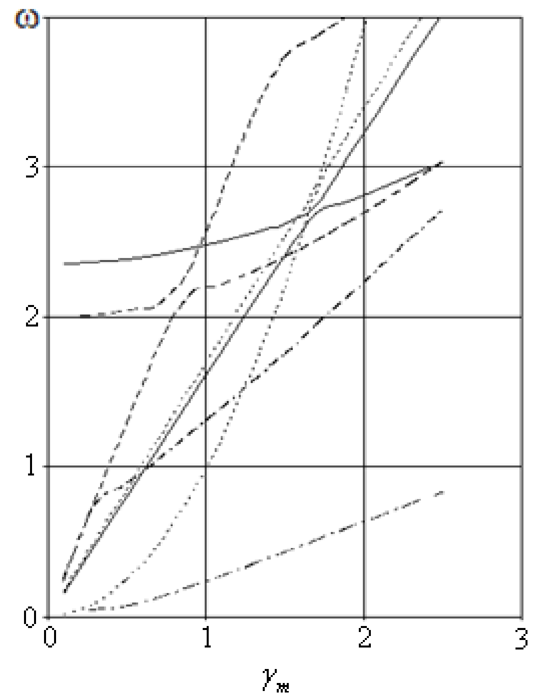


Fig.2. Natural frequencies comparison; **Fig.3**. Comparison of the natural longitudinal-radial vibration frequencies based on different theories (9) and (10).

- From the graphs in Fig. 2 and Fig. 3, it can be concluded that equations (1) accurately describe the wave process, similar to equations (11), for long shells ($l >> m\pi$), regardless of the values of the number m, i.e., for both low and high wave formation modes;
- Equations (1) are suitable for solving dynamic problems in shells of medium length with relatively low waveforms.
- These equations are not suitable for describing wave processes in short shells, whose lengths are comparable to the transverse dimensions of the shells $l >> m\pi$.

References

1. V T Grinchenko, & G L Komissarova 1980 Soviet Applied Mechanics 16 1–5.
2. B F Yalgashev 2007 Reports of the Academy of Sciences of the Republic of Uzbekistan 2 47-51.
3. V T Grinchenko, & V V Meleshko 1981 Kyiv: Naukova Dumka 283. [in Rus.].
4. G Herrmann, I Mirsky 1956 Journal of Applied Mechanics 23(4), 563-568.
5. I G Filippov, & K Kudainazarov 1990 Soviet Applied Mechanics 26(2) 161–168. doi:10.1007/bf00887110.
6. E I Grigolyuk, & I T Selezov 1973 Results of science and technology Series Deformable Solid Mechanics 5 Moskov: VINITI [in Rus.].
7. I G Filippov, & S I Filippov 2007 Moskov: Publishing house of MGSU 430.

8. I G Filippov, & K Kudainazarov 1990 Soviet Applied Mechanics 26(4) 351–357. doi:10.1007/bf00887127.
9. Kh Khudoynazarov 2003 Tashkent: Publishing House named after Abu Ali ibn Sino 325. [in Rus.].
10. I G Filippov, & K Kudainazarov 1998 International Applied Mechanics 34(12) 1204–1210. doi:10.1007/bf02700874.
11. A V Netrebko, S G Pshenichnov 2015 Problems of strength and ductility 77(1) 67-74/
12. Kh Khudoynazarov, Sh M Burkutboyev 2017 Mathematical Modeling and Numerical Methods 16 31–47. https://doi.org/10.18698/2309-3684-2017-4-3147.
13. Kh Khudoynazarov, A Abdirashidov, Sh M Burkutboyev2016 Mathematical Modeling and Numerical Methods 9 38–51.
14. R I Khalmuradov, Kh Khudoynazarov 2002 The 7th Conference "Shell Structures, Theory and Applications", Gdansk-Jurata (Poland) 131-132.
15. Kh Khudoynazarov, B F Yalgashev 2016 Scientific and technical journal Problems of Architecture and Construction 4 119-125. [in Rus.].
16. Markus Stefan 1988 Amsterdam: Elsevier 195p.
17. Sh H Arshad, at all 2011 Archive of Applied Mechanics 81 319–343. DOI 10.1007/s00419-010-0409-8
18. Ялгашев, Б. (2021). Самарқанд миллий меъморчилик саънатининг ўзига хос жиҳатлари (имом мотурдий мақбараси мисолида). Глобальное партнерство как условие и гарантия стабильного развития, 1(1), 448-450.
19. Abdikarimov R A, Khudayarov B A 2019 International Applied Mechanics 50(4) 389-398. https://DOI 10.1007/s10778-014-0642-x
20. Khudayarov B A, Turaev F Zh 2019 Applied Mathematical Modelling 66 662-679 https://doi.org/10.1016/j.apm.2018.10.008.
21. J O Kim, H Y Chun Journal of Vibration and Acoustics 125(1) 39– 45. doi: https://doi.org/10.1115/1.1525004.
22. J Kudlička 2006 Journal of Sound and Vibration 294 (1–2) 368-373.
23. https://doi.org/10.1016/j.jsv.2005.10.011.
24. P Shakeri Mobarakeh, V T Grinchenko, & B E Soltannia 2018 Strength of Materials 50, 376–386. https://doi.org/10.1007/s11223-018-9981-x
25. V Grinchenko 2003 Journal of Engineering Mathematics 46, 281–297. https://doi.org/10.1023/A:1025049526317
26. W Abassi, A El Baroudi, F Razafimahery 2016 International Journal of Applied Mechanics 08(03) DOI: 10.1142/S1758825116500320
27. Li Xuebin 2008 Journal of Sound and Vibration 311(3–5) 667-682. doi.org/10.1016/j.jsv.2007.09.023.
28. N G Chetayev 1955 Moskow: Publishing house of GITTL 208.