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MATHEMATICAL MODELING OF HEAT SUPPLY SYSTEMS

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Abstract
This article highlights the importance of mathematical modeling methods in order to increase the energy efficiency of heat supply systems in buildings and structures. To reduce heat energy consumption, the possibilities of analyzing the temperature regime and simulating heat transfer processes are analyzed based on various physical and mathematical equations. Also, ways to achieve an optimal energy balance are considered, taking into account the thermal parameters of construction structures.

Keywords: Heat supply, mathematical modeling, energy efficiency, heat balance, heat transfer, construction structure, temperature regime, simulation, convection, thermal coefficient.

Introduction

Despite the fact that the cost of energy carriers stands, heat and electricity tariffs are increasing. On the basis of these changes, interest in energy efficient and resource-efficient technologies is growing by the state and by heaters. In the world energy consumption balance, about 40% of thermal energy is at the expense of buildings and structures, 50% of this energy is sorted into maintaining a microclimate corresponding to human activities in the world. In this regard, the energy supply of buildings and structures is gaining a certain acuity. Today, the world's advanced engineering centers (ASHRAE.ISO, ABOK, and others) focused on developing standards for energy efficient buildings and structures. An important factor in reducing energy consumption is the development of special programs. High requirements are imposed on the development of such programs [1].

Factors affecting model adequacy are: detailing the barrier structures model, modeling engineering systems, microclimate factors (cycle, heat loads, humans).

Existing trends in the field of energy and resource conservation require the development and acquisition of new structures, technologies and engineering equipment. The most important direction is to increase the thermal coefficient of structures on the fence, a method that allows you to reduce the energy loss of residential buildings and, through it, the energy consumption for heating.

When laying the thermal regime, the most important properties of the structure and the physical processes that occur were taken into account, that is, to some extent, an idealized thermal model was created.

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The basic requirement for the heat model can be expressed as follows: the heat model must correspond to the phenomena under study and be implemented mathematically. There are different ways to tax heat transfer costs. Their main types are expert engineering and analytical methods. Currently, the analytical method is considered one of the most promising, since there are high-performance computer techniques.

For this reason, the task of developing an analytical and numerical model that can be implemented in universal computing techniques and applied in production conditions is relevant.

Heat transfer in a body can be written with the following equation, taking into account convection:

$$\rho c_p \frac{\partial T}{\partial t} + \nabla\left(-\chi \nabla T\right) = Q + \rho c_p \cdot u \nabla T ; \qquad (1)$$

Here: c_p – isobaric heat capacity, $\left[J/\left(kg\cdot {}^\circ C\right)\right]$;

ρ – material density, $\left[kg/m^3\right]$;

T – temperature, $\left[{}^\circ C\right]$;

u – the area of velocities, $\left[m/c\right]$;

Q – capacity of internal heat sources, $\left[W/m^2\right]$;

χ – hot conductivity coefficient, $\left[W/\left(m\cdot {}^\circ C\right)\right]$;

t – time, [seconds].

(1) – the expression describes the process of hot transfer at any point in the body. Methods of mathematical modeling the thermal balance of buildings make it possible to assess the main peculiarities of heat supply systems, taking into account the thermal – physical and mathematical parameters of the territory.

The dynamic model of the heat balance of the house takes into account the interaction between heating systems and heat exchange, as the internal and external climate is. It makes it possible to predict temperature changes in the room due to various factors, such as heat losses of thermal loads radiation heat.

The heat transfer coefficient of a building wall can be calculated by the following formula:

$$k = \frac{1}{\frac{1}{\alpha_1} + \sum \frac{\delta_i}{\chi_i} + \frac{1}{\alpha_2}} ; \qquad (2)$$

Here: α_1 – coefficient of heat transfer from the inner surface of the wall, $\left[W/\left(m^2\cdot {}^\circ C\right)\right]$;

δ_i – layer thickness of the wall;

χ_i – thermal conductivity of the wall layer coefficient, $\left[W/\left(m\cdot {}^\circ C\right)\right]$;

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α_2 – coefficient of heat transfer from the outer surface of the wall, $\left[W/\left(m^2\cdot°C\right)\right]$.

Thus, the mathematical model of temperature makes it possible to correct the air regime of buildings and structures depending on the changes in solar radiation and external temperature.

G.Liang, H.Zhang and Ch.Ding paper analyzed energy flows in emergencies that occur in systems where electricity and thermal energy are integrated, especially in the presence of a ring-shaped heat supply network. The study envisages the study of how the reliability and stability of such integrated systems change in emergency situations through mathematical models and simulation. The authors took into account real-time monitoring, system equilibrium, heat flow directions, and Electro-Thermal conversion efficiency in energy flow analysis. In particular, the possibility of providing heat supply with alternative routes through the heat Ring network has been shown to be an important factor in keeping the system stable in case of emergency. Heat supply networks are one of the complex engineering systems with a high degree of hydraulic conductivity. The graph theory that operates on the current and voltage laws of Kirchhoff applied in electrical networks can also be extended to thermal networks. An accident or failure in the hydraulic state that occurs in any pipe piece in these networks also causes changes in hydraulic parameters such as current (mass consumption) and pressure in other network parts. This directly affects the overall stability and efficiency of the system [6].

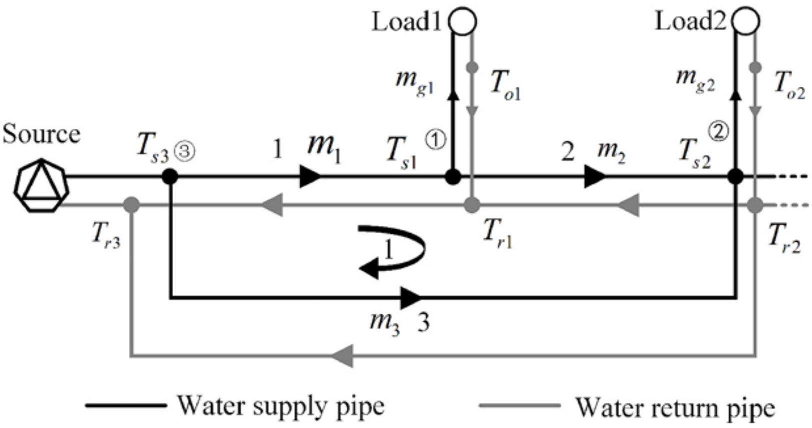

Figure 1. Schematic diagram of a single-contour thermal network

T_s, T_r – water temperature at different points; m_1, m_g – mass flows;

Based on the standard "ANSI/ASHRAE Standard 90.1-2016" developed by ASHRAE, it is used to assess the energy efficiency of buildings. The table lists annual energy consumption, source energy consumption, energy cost index, carbon emissions and social spending ($SC - CO_2$) by different building types [1,2].

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Table 1.

Type of building	Model building	Area share (%)	Annual energy consumption ($kW/m^2 \cdot$ year)	Source energy consumption ($W/m^2 \cdot$ year)	Energy cost index ($\$/m^2 \cdot year$)	Carbon exhaust ($tonne/m^2 \cdot$ year)	The social cost of carbon waste $SC-CO_2$ ($\$/m^2 \cdot year$)
Office	Small office	3.8%	27.1	77.6	$0.82	5.5	$275
	Middle office	5.0%	30.8	84.2	$0.88	5.9	$296
	Big office	3.9%	55.4	156.9	$1.65	11.1	$555
Trade	Solo department store	10.9%	48.4	114.4	$1.15	7.8	$389
	Mall	3.7%	52.8	133.8	$1.37	9.2	$462
Education	Primary school	4.8%	43.4	107.4	$1.09	7.4	$369
	High school	10.9%	37.2	94.0	$0.96	6.5	$325
Health	Outpatient	3.4%	107.6	276.3	$2.84	19.1	$958
	Hospital	4.5%	120.0	276.8	$2.77	18.7	$936
Guest-room	Small hotel	1.6%	54.8	118.0	$1.16	7.8	$392
	Grand Hotel	4.2%	83.1	177.1	$1.73	11.7	$586
Warehouse-room	Non-cooling barn	18.6%	15.7	33.2	$0.32	2.2	$110
Food service	Fast service restaurant	0.3%	493.4	863.7	$7.87	53.7	$2,689
	Full service restaurant	1.0%	336.5	649.8	$6.14	41.7	$2,090
Accommodation	Middle story house	13.7%	37.8	104.4	$1.09	7.3	$367
	Apartment building	9.6%	41.3	92.0	$0.91	6.2	$308
National average	—	100%	48.6	116.0	$1.17	7.9	$395

This norm is considered one of the main technical documents that assess energy efficiency in the United States and internationally. It serves as an important methodological basis in the planning, modeling and optimization of heat supply systems.

In the management and modeling of modern Heat supply systems, high accuracy is required, especially in multi-zone buildings, the thermal condition of each area varies. While traditional approaches to ordinary differential equations are focused on describing changes in time, they do not fully account for changes in location. This does not provide sufficient accuracy for systems with complex thermal dynamics.

Ordinary differential equations (ODE) is an equation in which derivatives of a function dependent on one or more variables are involved. It is written with respect to only one independent variable (usually time t). For example:

$$\frac{dy}{dt} + ky = 0 \qquad (3)$$

Here: $y(t)$ - time-dependent variable (temperature),

k - constant coefficient,

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t - time.

ODE is widely used in heat transfer, electrical circuits, dynamical systems and other industries.

For this reason, models based on partial differential equations (PDE – Partial differential equations) are increasingly widely used. In this approach, the temperature distribution will depend not only on time, but also on spatial coordinates (x, y, z). This makes it possible to more accurately model mechanisms such as thermal conductivity, convection, radiation [7]. As an example below, the thermal conductivity equation:

$$\frac{\partial T}{dt} = \alpha \cdot \frac{\partial^2 T}{\partial x^2} \qquad (4)$$

Bu yerda: $T(x,t)$ - temperature, location (x) and time (t) function,

$\frac{\partial T}{dt}$ - change in time (derivative of temperature by time),

$\frac{\partial^2 T}{\partial x^2}$ - spatial second order derivative (spatial derivative of temperature gradient),

α - heat diffusion coefficient (depends on material property).

The PDE – mathematical zonal control model proposed by He and Gonzalez (2015) is one of the advanced approaches in the field. They have developed a control strategy in the HVAC (Heating, ventilation, and air conditioning) system that focuses on minimizing energy consumption by linking zonal models to controls to keep the temperature of each zone at an optimal level. The model takes into account the following aspects:

1. Each zone (room, compartment) has its own heat load, size and external conditions.
2. Zonal temperature variations are characterized by convection-diffusion type PDEs.
3. Individual control signals (e.g. radiator output, fan speed) are detected for each zone.

With the help of such an approach, the thermal needs of different zones are analyzed independently, which allows you to control the individual microclimate, increase comfort, and save energy. Integration of PDE modeling into zone management:

1) in modeling, parameters such as temperature radius, heat flow, speed of movement of air are determined.

2) the model is simulated by numerical methods (e.g. numerical differential solutions, bounded element method).

3) while the optimal management strategy is developed through the adjoint method or dynamic programming methods.

It is considered relevant in multi-stored buildings, open-office settlements or climate-friendly zoned-controlled buildings. This determines the optimal temperature for each area (zone) through the built-in zonal control [8]. ODE models instead provide a high-resolution description Through Spatial and time-dependent convection-diffusion equations.

Mathematical formula of the model. The most general formula is based on the equation of convection and thermal diffusion:

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$$\rho c_P \frac{\partial T(x,t)}{\partial t} + \rho c_P \upsilon(x,t) \cdot \nabla T(x,t) = k \Delta T(x,t) + Q(x,t) \qquad (5)$$

Here: $T(x,t)$ - temperature in place and time,

ρ - air density,

c_P - heat capacity,

$\upsilon(x,t)$ - air flow rate,

k - heat diffusion coefficient,

$Q(x,t)$ - heat source (fan, radiator, etc).

PDE model zonal control. Heat transfer is represented by the following generalized differential equation:

$$\rho c_P \frac{\partial T(x,t)}{\partial t} = k \frac{\partial^2 T(x,t)}{\partial x^2} + Q(x,t) \qquad (6)$$

Here: $T(x,t)$ - function of temperature by space and time.

This equation is used to determine the heat distribution for multilayer walls or multi-zone buildings [9].

Conclusion

Mathematical modeling of Heat Supply Systems is recognized as the main means of generating economic and environmental benefits not only by ensuring energy efficiency, but also by rational use of available resources, identification of heat losses and their minimization. In the process of such modeling, the geometric and physical properties of the building, factors of the external environment, modes of operation of heat sources and the requirements of heat consumers are taken into account in a complex way. A properly structured mathematical model allows for an optimal heat distribution for each room or zone of a building, which serves to reduce operating costs by reducing excess energy consumption. Modeling also predicts failures or decreased efficiency that occur in the operation of heat supply systems, and effective measures for their elimination are developed. This not only increases the energy efficiency of buildings, but also serves the principles of sustainable development, reducing their negative impact on the environment.

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