

# MATHEMATICAL METHODS OF DECISION MAKING UNDER UNCERTAINTY IN THE HOTEL BUSINESS

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## Abstract

This article analyzes various management decision-making processes in the hotel industry for hotel managers at various levels under conditions of insufficient or unreliable information, and examines decision-making mechanisms using various mathematical criteria. Flexible algorithms for making management decisions are proposed.

**Keywords:** Management decision-making, data uncertainty, unreliable data, uncertain inputs, evaluation criteria.

## Introduction

Currently, the global tourism market is characterized by a high level of competition, and a specialist working in the field of tourism and hotel business must be ready to make decisions in an uncertain competitive environment.

Management of the hotel business in a market economy is characterized by many factors of uncertainty and risk. These factors are greatly enhanced in times of crisis and economic instability, when most decisions are made in conditions of constantly changing environmental factors. Decision-making can be viewed as a problem of analyzing complex systems. When making decisions, modern leaders, functional managers and economic analysts must rely not only on experience and intuition, but, above all, on scientific decision-making methods.

The mathematical theory of decision making allows specialists to better understand the problem and avoid unnecessary mistakes. In decision theory, there is a special term: decision maker. This is a management subject, which can be a manager, economist, entrepreneur, executive or a group of individuals.

The decision maker interacts with the external environment and understands various possible environmental situations. The uncertainty associated with the possible negative consequences of a decision is described by the concept of risk.

Risk is an activity associated with overcoming uncertainty in a situation of inevitable choice, in which the probability of achieving the desired result can be quantitatively and qualitatively assessed. Risk is associated with the choice of certain alternatives. A solution with less risk can be chosen, but the resulting benefit will also be lower. The benefit with the highest risk has a higher value.

In the process of making a decision under uncertainty, the decision maker must determine the goal of the decision, find possible options for solving the problem, calculate the possible outcomes of each decision, evaluate each outcome, and choose the optimal solution based on the established goal and the level of acceptable risk (Table 1).

The following criteria are discussed here, which are recommended for making decisions based on the level of risk.

*Table 1*

Risk Level	Recommended criterion
Extreme Caution	Wald's rule
Minimal Risk	Savage's rule
Compromise in Solution	Hurwitz's rule
(Depends on Pessimism Coefficient)	Laplace's rule

The choice of decision criteria is the most difficult and crucial step. There are no general recommendations. If a very important decision is being made and even minimal risk is not acceptable, then the Wald rule - a guaranteed result - should be used. If a certain risk is acceptable, then the Savage rule is chosen. From the point of view of the theory of mathematical modeling, decision making means solving an extremum problem, that is, finding the extremum of a function called the objective function under certain constraints

Let us assume that a decision maker is considering several possible solutions  $i=1, \dots, m$ . The situation is uncertain, only one of the options  $j=1, \dots, n$  is certain. If the  $i$ -th decision is made and the situation is  $j$ -th, then the firm headed by the decision maker will receive a profit  $q_{ij}$ . The matrix  $Q=(q_{ij})$  is called the matrix of consequences (possible decisions).

Here we consider a mathematical model of decision making under uncertainty using an example from the hotel business

### LITERATURE ANALYSIS

In order to increase the efficiency of the information systems of hotel complexes, artificial intelligence methods and tools that allow to improve the quality of decisions made in conditions of uncertainty of the purpose, functioning and manifestation of the external environment play an important role. [1-3]. Sources of uncertainty may include inconsistencies or contradictions between the requirements of planned actions and allocated resources.

Until recently, it was believed that there were two main approaches to making management decisions in conditions of uncertainty: deterministic and probabilistic. In the last decade, a third, fundamentally new approach based on fuzzy multi-characteristics has appeared [6-7].

In the formalization of the problem, an attempt is made to reduce the level of uncertainty by involving additional information. In fact, a transition from complete uncertainty to stochastic uncertainty or fuzzy sets is made, which leads to a set of permissible quasi-optimal solutions. The selection of the best of the quasi-optimal solutions is carried out based on various



intelligent procedures for making management decisions under a certain type of uncertainty [4, 5].

Most management decision-making tasks must be solved in conditions of uncertainty - incompleteness, unreliability and untimely receipt of information. There are many sources and forms of manifestation of uncertainty [4,8]:

- 1) uncertainty caused by lack of information and its reliability;
- 2) uncertainty associated with time constraints for making a decision;
- 3) uncertainty caused by the high price paid for its accuracy;
- 4) uncertainty arising from the inconsistency of the model due to the ambiguity of the purpose and criteria of operation, the ambiguity of the manifestation of the external environment relative to the system under consideration, uncertainty in choosing the reasonable complexity of the model;
- 5) uncertainty caused by the actions of people in the decision-making process;
- 6) uncertainty deliberately created to hide the behavior of the system and its resources.

Currently, the personal factor in making management decisions is becoming one of the sources of significant uncertainty, ambiguity and uncertainty in the description of many important technical tasks [10]. L.A. Zadeh developed a fuzzy set apparatus that allows solving classical mathematical problems with significant uncertainty and fuzzy logic, overcoming the problem of subjectivity [6-7].

## RESEARCH METHODOLOGY

The research used scientific abstraction, grouping, comparison, empirical analysis, and other methods.

## ANALYSIS AND RESEARCH RESULTS

**Problem Statement.** An entrepreneur wants to lease a hotel for a year. There are four types of hotels: 20, 30, 40, and 50 rooms. According to the terms of the lease, the entrepreneur must cover all operating costs. These costs (in various currencies) are listed in Table 2. The entrepreneur's revenue is \$200 per night for each occupied room. The goal is to determine the optimal size of the hotel to lease to maximize profit.



Table 2

Entrepreneur's expenses	Number of rooms			
	20	30	40	50
1. Costs not related to the choice of hotel project (in US dollars per year)				
a) landscaping costs	10 000	12 000	20 000	40 000
b) current hotel repair and maintenance costs	1500	3000	6000	10 000
c) security staff salaries	6000	6000	12 000	12 000
d) reception and accommodation staff salaries	8000	8000	14 000	16 000
d) rent	200 000	300 000	400 000	500 000
2. Costs by number of hotel rooms (US\$ per year)				
a) Room furnishing costs (furniture, plumbing, etc.)	2000	2000	2000	2000
b) Maintenance costs per room	150	150	150	150
Business expenses	Xonalar soni			
c) Fire insurance per room	20	30	40	50
d) Housekeeping costs (1 room per 10 rooms)	25	25	25	25
2. Costs by number of hotel rooms (US\$ per year)	600 0	600 0	600 0	6000
3. Expenses (cubic meters per day) based on the number of occupied rooms				
a) household expenses (laundry, cleaning)	25	25	25	25
b) utility bills (electricity, water supply, etc.)	25	25	25	25

The solution to the problem begins with the creation of the outcome matrix. Here, the alternatives are hotel types, so we can take  $I = \{20, 30, 40, 50\}$  as a set of alternatives. In the example under consideration, the outcome can be considered as the profit that the entrepreneur receives for renting a hotel for one year. For a given alternative  $i \in I$ , its profit is determined by the average number of occupied rooms  $J$  (i.e., the average annual demand is the only parameter characterizing the state of the environment here). Therefore,  $J = \{1, 2, \dots, 50\}$  can be taken as the set of situations in this problem. The objective function  $q$  here has the following meaningful meaning:  $q_{ij}$  is the profit that the entrepreneur receives for one year in the case of renting a hotel with  $i$  rooms and  $J$  is the average annual demand. We find the objective function in the exact form.

The non-intelligible costs are:

$$10,000 + 1,500 + 6,000 + 8,000 + 200,000 = 225,500 \text{ (for 20 rooms),}$$

$$12,000 + 3,000 + 6,000 + 8,000 + 300,000 = 329,000 \text{ (for 30 rooms),}$$

$$20,000 + 6,000 + 12,000 + 14,000 + 300,000 = 452,000 \text{ (for 40 rooms),}$$

$$40,000 + 10,000 + 12,000 + 16,000 + 500,000 = 578,000 \text{ (for 50 rooms).}$$

The costs, depending on the number of hotel rooms, are

$$2000i + 6000/10i + 150i + 25i = 2775i.$$

The costs, depending on the number of occupied rooms  $J$ , are as follows:

$$365(25j + 25j) = 18250j.$$

The entrepreneur's income is determined by the number of occupied rooms  $J$  and is equal to:

$$365 \times 200j = 73\,000j.$$

Therefore, the profit of the entrepreneur in position  $(i, j)$  for the year is:

$$q_{ij} = \begin{cases} 73\,000j - (2775i + 18250j + 225500), & j = 1, \dots, 20 \\ 814\,000, & j = 21, \dots, 50 \end{cases}$$



(for 20 rooms),

$$q_{ij} = \begin{cases} 73\,000j - (2775i + 18\,250j + 329\,000), & j = 1, \dots, 30 \\ 123\,0250, & j = 31, \dots, 50 \end{cases}$$

(for 30 rooms)

$$q_{ij} = \begin{cases} 73\,000j - (2775i + 18\,250j + 452\,000), & j = 1, \dots, 40 \\ 1627000, & j = 41, \dots, 50 \end{cases}$$

(for 40 rooms)

$$q_{ij} = 73\,000j - (2775i + 18\,250j + 578\,000), \quad j = 1, \dots, 50$$

(for 50 rooms).

Thus, the table of consequences has the following form (Table 3).

Table 3

Hotel type	0	1		50
20 rooms	-281 000	-226 250		814 000
30 rooms	-412 250	-357,500		1 230 250
40 rooms	-563 000	-508 250		1,627,000
50 rooms	-716 750	-662 000		2,020,750

What decision should the decision maker make? In a situation of complete uncertainty, only a few initial recommendations can be made. These recommendations will certainly not be accepted. Much will depend, for example, on their risk appetite. But how can we assess the risk in this scenario? Suppose we want to assess the risk posed by the *i*-th decision. We do not know the true situation. But if we did, we would choose the best decision, that is, the one that would yield the highest return. In other words, if the situation were *j*-th, then the decision that would yield  $q_j = \max q_{ij}$  return would be made.

This means that by making the *i*-th decision, we run the risk of getting only  $q_{ij}$ , not  $q_j$ . This means that making the *i*-th decision carries the risk of not getting  $r_{ij} = q_j - q_{ij}$ . The matrix  $R=(q_{ij})$  is called the risk matrix. In the example of hotel rental, the risk matrix is given in Table 4..

Table 4

Hotel type	0	1	50
20 rooms	0	0	1,206,750
30 rooms	131 250	131 250	790 500
40 rooms	282 000	282 000	393 750
50 rooms	435 750	435 750	0

It can be noted that here it is necessary to assess the quantitative risk. Some guidelines for decision-making can be the following [8,9,10,11,12].



1. Wald's rule (the rule of extreme pessimism). When considering the  $i$ -th solution, we assume that the situation is actually the worst, that is, it brings the least profit.

$$a_i = \min_j q_{ij}$$

We choose the solution  $i_0$  with the largest  $a_i$ . Thus, Wald's rule recommends making the following decision:

$$a_{i_0} = \max_i a_i = \max_i \left( \min_j q_{ij} \right)$$

So, in the example above, we have:  $a_1 = -281,000$ ,  $a_2 = -412,250$ ,  $a_3 = -563,000$ , and  $a_4 = -716,750$ . Now we find the maximum value of the numbers  $a_1$ ,  $a_2$ ,  $a_3$ , and  $a_4$ . This is  $a_1$ . This means that Wald's rule recommends renting a 20-room hotel.

2. Savage's rule (minimum risk rule). When applying this rule, the risk matrix  $R = (r_{ij})$  is analyzed. Considering the  $i$ th solution, we assume that the situation with maximum risk  $b_i = \max_j r_{ij}$  is actually developing. But now we choose the solution  $i_0$  with the smallest  $b_i$ . Thus, Savage's rule recommends making a decision equal to  $i_0$ :

$$b_{i_0} = \min_i b_i = \min_i \left( \max_j r_{ij} \right)$$

So, in the example above, we have  $b_1 = 1,206,750$ ,  $b_2 = 790,500$ ,  $b_3 = 393,750$ ,  $b_4 = 435,750$ . Now we find the minimum of the numbers  $b_1$ ,  $b_2$ ,  $b_3$ ,  $b_4$ . This is  $b_3$ . This means that Savage's rule recommends renting a 40-room hotel.

3. Hurwitz's rule (weighing pessimistic and optimistic approaches to a situation). The decision that achieves the maximum result is  $i$  - taken

$$\left\{ \lambda \min_j q_{ij} + (1-\lambda) \max_j q_{ij} \right\}$$

Here  $0 \leq \lambda \leq 1$ . The value of  $\lambda$  is called the pessimism coefficient and is chosen based on subjective considerations. If  $\lambda$  approaches one, the Hurwitz rule approaches the Wald rule, and when  $\lambda$  approaches zero, the Hurwitz rule approaches the "pink (extreme) optimism" rule. In the example, for  $\lambda = 0.5$ , we obtain the following values (Table 5).

Table 5

Number of	$\min_j q_{ij}$	$\max_j q_{ij}$	$\left\{ 0,5 \min_j q_{ij} + (1-0,5) \max_j q_{ij} \right\}$
20	-281 000	814 000	266 500
30	-412 250	1 230 250	409 000
40	-563 000	1,627,000	532 000
50	-716 750	2,020,750	652 000

Therefore, in this case, Hurwitz's rule recommends renting a 50-room hotel. For  $\lambda = 0.74$ , we obtain the values given in Table 6.

Table 6

Number of rooms	$\min_j q_{ij}$	$\max_j q_{ij}$	$\{0,74 \min_j q_{ij} + (1-0,74) \max_j q_{ij}\}$
20	-281	814 000	3700
30	-412	1 230 250	14 800
40	-563	1,627,000	6400
50	-716	2,020,750	-5000

Therefore, in this case, the Hurwitz rule recommends renting a 30-room hotel.

1. Laplace's rule (equal probability rule), when the probabilities of all situations are considered to be equal to  $-1/n$ . After that, one of the following two decision-making rules can be chosen. The rule of maximizing the average expected profit. The average expected profit received by the firm is determined by implementing the  $i$ -th solution and by the following formula:

$$M[Q_i] = \frac{1}{n} \sum_{j=1}^n q_{ij}$$

As a rule, it is recommended to make a decision that will bring the maximum expected average revenue.

In the example, the maximum expected average revenue corresponds to solution 3, that is, in this case it is best to rent a hotel with 40 rooms (Table 7).

Table 7

Hotel type	0	1	50	$M[Q_i] = \frac{1}{n} \sum_{j=1}^n q_{ij}$
20 rooms	-281 000	-226 250	814 000	588 558.8
30 rooms	-412 250	-357,500	1230 250	731 058.8
40 rooms	-563 000	-508 250	1,627,000	746 705.8
50 rooms	-716 750	-662 000	2,020,750	652 000

The rule of minimizing the average expected risk. The average expected risk of the firm when implementing the  $i$ -decision is determined by the following formula:

$$M[R_i] = \frac{1}{n} \sum_{j=1}^n r_{ij}$$

The rule recommends making a decision that involves a minimum average expected risk. In the example given, we calculate the average expected risks.



Table 8

Hotel type	0	1	50	$M[R_i] = \frac{1}{50} \sum_{j=1}^{50} r_{ij}$
20 rooms	0	0	1,206,750	340 147.0
30 rooms	131 250	131 250	790 500	197 647
40 rooms	282 000	282 000	393 750	182 000
50 rooms	435 750	435 750	0	276 705.8

The minimum average expected risk corresponds to solution 3, that is, in this case it is best to rent a 40-room hotel (Table 8).

In the example under consideration, we present possible options for making a decision depending on the level of risk (Table 9).

Table 9

Risk level	Recommended criterion	Hotel type
Extremely cautious	Wald's rule	20 rooms
Minimal risk	Savage's rule	40 rooms
Pessimism coefficient $\lambda = 0.5$	Hurwitz's rule	50 rooms
Pessimism coefficient $\lambda = 0.74$		30 rooms
Maximum average expected return	Recommended criterion	40 rooms
Minimum average expected risk		40 rooms

## CONCLUSION

The methods considered in the article demonstrate the possibilities of using mathematical models and assumptions in analyzing the possible consequences of decision-making. The hotel industry is characterized by a high level of risk due to the seasonal nature of demand and a high level of dependence on capital investments. Understanding the principles of optimal behavior under uncertainty creates the necessary scientific basis and allows for a certain degree of optimization of management decision-making.

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