DYNAMIC STABILITY AND FREE VIBRATIONS OF A SHAFT MOUNTED ON VISCOELASTIC SUPPORTS: PROBLEM FORMULATION AND SOLUTION METHODOLOGY

Navrozbek Qalandarov 1
Muhammad Narzulloyev 2
1Asia International University
2 Zarafshan Polytechnic College No. 1

Abstract
This paper addresses the dynamic stability and free vibrations of a mechanical system with a finite number of degrees of freedom, consisting of a deformable shaft mounted on viscoelastic supports. The mathematical formulation of the problem is based on the principle of possible displacements, from which a general variational equation is derived. The rheological properties of the deformable supports and the shaft are described using the linear hereditary Boltzmann-Volterra relation together with the weakly singular Koltunov-Rzhanitsyn kernel. Owing to the smallness of the integral terms in the constitutive relation, the "freezing" method is applied, and the transcendental equation determining the free-vibration frequencies of the system is solved numerically by the Müller method. The dynamic stability of the rotating shaft-support system is assessed using the D-partition method. The obtained results are compared with known solutions by A.S. Kelzon and V.I. Yakovlev, the discrepancy not exceeding 15%.

Keywords: Viscoelastic support, shaft dynamics, free vibrations, critical speed, Koltunov-Rzhanitsyn kernel, freezing method, dynamic stability.

Introduction

Shaft structures mounted on supports are widely used in mining, mechanical engineering, aviation and space industries, as well as in the food and transport sectors. Ensuring the reliability of such structures requires keeping the dynamic stresses and deformations arising in the shafts within permissible limits and developing reliable calculation methods that guarantee the most favorable distribution of accumulated stresses. Leading research centers in the USA, France, Japan, Italy, Russia, China, Turkey and other countries devote considerable attention to this field.

A review of the literature shows that a mechanical system consisting of a support and a shaft with a finite number of degrees of freedom, mounted on viscoelastic supports, has not received sufficient attention as a non-homogeneous dissipative mechanical system. In particular, a general approach to determining force factors in various types of shaft structures, evaluating the influence of inertial forces, and establishing stability conditions that account for the viscosity of the supports has not yet been fully developed — a gap that is especially significant for machines operating at medium and high speeds. To address this gap, the present paper introduces a mathematical model, solution methodology and algorithm for studying the dynamic stability and free vibrations of a shaft mounted on viscoelastic supports.

The purpose of this research is to determine the critical speed of a shaft structure mounted on viscoelastic supports, to assess its dynamic stability, and to develop a methodology for determining the frequencies and mode shapes of free vibrations.

The object of the study is the deformable shaft and support structures, while the subject is the theory and solution methods for determining the critical speed and the dynamic stress-strain state, taking into account the rheological properties of the deformable shaft materials under dynamic loading. The research employs methods of deformable solid mechanics and structural mechanics, computational mathematics, mathematical modeling, and, for solving partial differential equations, the freezing method, separation of variables, the Gauss method, the Laplace method and the finite-element method.

The scientific novelty of the study consists in the formulation of the problem of dynamic stability analysis and determination of the stress-strain state of a shaft mounted on viscoelastic supports, together with the development of the corresponding solution methodology and algorithm; in the determination of the critical speed of such a shaft structure and a comparative assessment of its stability; and in the development of a methodology and algorithm for studying the free vibrations of the deformable shaft, including the determination of the vibration frequency and construction of the mode shape.

2. MATERIALS AND METHODS: MATHEMATICAL FORMULATION

Consider a rotor system consisting of N rigid and K deformable viscoelastic elements. The system contains S support elements, of which S_1 are deformable bearings and S_2 are massless deformable elements (springs). The problem is formulated using the principle of possible displacements: the sum of the variations of the work done by the active and passive forces acting on the mechanical system must equal zero:

$$\delta A = \delta A\_b + \delta A\_u + \delta A\_F = 0 \qquad (1)$$

where δA_b, δA_u and δA_F denote the variation of the work done by the stiffness, inertial and external forces, respectively.

For the elements of the mechanical system, the relationship between stress σ and strain ε is described by the linear hereditary Boltzmann-Volterra relation:

$$\sigma_{ij}(t) = E_0 \left[\varepsilon_{ij}(t) - \int_0^t \Gamma(t-\tau)\, \varepsilon_{ij}(\tau)\, d\tau\right] \qquad (2)$$

where E_0 is the instantaneous modulus of elasticity and Γ(t) is the relaxation kernel. The weakly singular Koltunov-Rzhanitsyn kernel and Rabotnov's fractional-exponential function are used as the relaxation kernel. Since the integral term in the constitutive relation (2), which characterizes the rheological properties of the deformable elements, has a small value, the "freezing" method can be applied. This yields the following complex stiffness relation for the massless deformable elements:

$$F_e = -c_e \Delta e = -c_e[1 - \Gamma^c_e(\omega R) - i\Gamma^s_e(\omega R)]\Delta e \qquad (3)$$

where c_e is the stiffness of the element, Δe is its displacement, Γ^c_e and Γ^s_e are the cosine and sine (Fourier) images of the relaxation kernel, and ωR is the real part of the complex frequency.

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Using Lagrange's equations of the second kind, the vibrations of the mechanical system are described by the following system of integro-differential equations:

$$M \ddot{q} + C(1 - \Gamma^c - i\Gamma^s)\, q = Q(t) \qquad (4)$$

where M is the mass matrix, C is the stiffness matrix, q is the vector of generalized coordinates, and Q(t) is the vector of external forces. For free vibrations, the right-hand side of equation (4) vanishes, and the solution is sought in the form:

$$q(t) = q_0 \cdot e^{i\Omega t} \qquad (5)$$

where Ω is the unknown complex frequency of free vibrations. Substituting (5) into (4) yields a system of homogeneous algebraic equations, whose condition for a non-trivial solution gives the transcendental equation determining the frequency:

$$\det[C(1 - \Gamma^c(\Omega) - i\Gamma^s(\Omega)) - \Omega^2 M] = 0 \qquad (6)$$

Equation (6) is solved iteratively by the Müller method, with the solution of the conservative problem (neglecting viscosity) used as the initial approximation. Since the shafts are curvilinear structures, their equation of motion is a system of partial-derivative integro-differential equations, forming a closed system of 12 equations with 12 unknowns. At the fixed end of the rod, the displacement and rotation are set to zero, while at the free end a boundary condition is imposed corresponding to the given amplitudes of the applied force and moment.

3. RESULTS: CRITICAL SPEED DETERMINATION AND STABILITY ANALYSIS

Forced harmonic vibrations are sought using the complex-amplitude method, which leads to a system of non-homogeneous algebraic equations solved by the Gauss method. To determine the critical speed of the rotating rotor system, the determinant composed of the coefficients of the characteristic equation is set equal to zero, and the resulting equation is solved by the Müller method. The dynamic stability of the rotor system is examined using the D-partition method: the stability region is bounded by the complex roots of the characteristic equation (22) for which the real part is equal to zero.

For the numerical analysis, a shaft made of 40St steel was considered, consisting of seven elements with an inner diameter of 0.055 m, an outer diameter of 0.070 m, and lengths ranging from 0.045 to 0.0515 m (Table 1).

Table 1. Geometric parameters of the shaft

Element	Nodes	Inner diameter, m	Outer diameter, m	Length, m
1	1-2	0.055	0.070	0.049
2	2-3	0.055	0.070	0.049
3	3-4	0.055	0.070	0.045
4	4-5	0.055	0.070	0.0515
5	5-6	0.055	0.070	0.0515
6	6-7	0.055	0.070	0.0480
7	7-8	0.055	0.070	0.0490

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Using the finite-element method, calculations were performed at 20 points along the shaft. The graphs of the vibration amplitude versus the external-force frequency, constructed on the basis of the critical speed, show that the displacement amplitude of the rotor mounted on the shaft reaches resonance at certain frequency values. It was found that the vibration amplitude in the vertical direction exceeds that in the horizontal direction by up to 25%, which is explained by the anisotropic viscoelastic properties of the supports.
The difference between the phase velocities varies within the range between two critical frequencies and remains constant outside this range. Accounting for the viscosity of the supports was found to act as a stabilization factor that accelerates dynamic equilibration and damps the system — that is, viscoelastic supports limit resonance amplitudes and enhance the dynamic stability of the system.

4. DISCUSSION

The developed mathematical model and solution algorithm offer several advantages for engineering design practice. First, accounting for the viscoelastic properties of the supports enables a more accurate prediction of the critical speed, which helps prevent emergency operating regimes when designing machines and mechanisms. Second, the stability regions constructed by the D-partition method allow an engineer to select the operating speed range so as to avoid resonance zones. Third, the fact that the real and imaginary parts of the frequency are expressed as monotonically increasing functions of the angular velocity of rotation provides a quantitative basis for the constructive selection of the damping properties of the supports.
Furthermore, the identified difference of up to 25% between the vertical and horizontal vibration amplitudes indicates that anisotropy must necessarily be taken into account when designing shaft-support systems. In such cases, it is recommended that the stiffness and damping parameters of the supports be selected separately for each direction. The developed methodology can, in particular, be applied to the calculation of shaft structures used in medium- and high-speed grinding, aviation-space and machine-building units, since it allows both rigid and deformable elements to be treated as a single dissipative mechanical system.
Directions for future research include generalizing the methodology to multi-disk rotors, shafts with non-uniformly distributed mass, and nonlinear rheological models. In addition, optimizing the number and arrangement of supports along the system with respect to the critical speed value is of particular scientific and practical interest.
The scientific significance of the obtained results lies in their substantial contribution to the further development of the theory of dynamics of shaft structures mounted on viscoelastic supports. The practical significance lies in the fact that the results make it possible to study new regularities in assessing the stress-strain state of such shafts under harmonic and non-stationary loading, while the developed methods and computational algorithms serve as tools for solving and investigating shaft-vibration problems in engineering practice.

5. CONCLUSION

A mathematical model for studying the dynamic stability and free vibrations of a shaft mounted on viscoelastic supports has been developed on the basis of a general variational principle. By accounting for rheological properties through the Boltzmann-Volterra relation and the Koltunov-Rzhanitsyn kernel, applying the freezing method, and employing the Müller and Gauss methods, a reliable methodology and algorithm for determining the critical speed and free-vibration frequency have been created. The developed methodology enables the dynamic characteristics of a shaft-support system to be predicted with high accuracy and provides a basis for its practical application in industrial equipment. The reliability of the results is confirmed by the correct formulation of the boundary conditions, the rigor of the derived mathematical expressions, and the consistent use of established solution methods, as well as by comparison with the known results of A.S. Kelzon and V.I. Yakovlev, the discrepancy not exceeding 15%. The scientific significance of the results lies in their substantial contribution to the theory of dynamics of shaft structures mounted on viscoelastic supports, while their practical significance lies in enabling the study of new regularities in assessing the stress-strain state of such structures under harmonic and non-stationary loading. The problem of forced vibrations and the results of practical implementation are presented in the author's subsequent paper.

REFERENCES

1. Nematov E.Kh., Qalandarov N.O., Abdullayev Y.B. Material and strength characteristics of shafts used in rotary milling machines // Journal of Science and Technology Development. — 2024. — No. 2. — P. 27-32.
2. Teshayev M.X., Nematov E.Kh., Qalandarov N.O. Calculation of kinematics of grinding shafts in flour production // Journal of Science and Technology Development. — 2024. — No. 2. — P. 32-41.
3. Teshayev M.X., Nematov E.Kh., Qalandarov N.O. Problems of dynamics of a shaft mounted on viscoelastic supports // Mechanics and Technology Scientific Journal. — Namangan, 2024. — No. 3(16). — P. 18-23.
4. Teshayev M.X., Nematov E.Kh., Qalandarov N.O. Calculation of the dynamics of a shaft mounted on viscoelastic supports // Mechanics and Technology Scientific Journal. — Namangan, 2024. — No. 3(16). — P. 24-30.
5. Nematov E.Kh., Qalandarov N.O. Calculating of determining force and speed of rotary shafts for grinding // Mechanics and Technology Scientific Journal. — Namangan, 2023. — No. 4(13). — P. 69-79.
6. Safarov I., Teshaev M., Axmedov Sh., Marasulov A., Qalandarov N. Dynamic stress-strain states in viscoelastic half-spaces from the effects of cylindrical inclusion loads // AIP Conference Proceedings 2467, 060024. — 2022. — P. 1-10.
7. Teshaev M., Qalandarov N., Berdiyeva S. Method of balancing a rotational shaft for grain grinding // AIP Conference Proceedings 3268, 020023. — 2025. — P. 24.1-23.6.
8. Nematov E.Kh., Qalandarov N.O. Calculation of the torque of the rotary shaft // Eurasian Journal of Research, Development and Innovation. — 2023. — Vol. 24. — P. 1-4.
9. Qalandarov N.O. Features of the dynamics of a shaft mounted on viscoelastic supports and the stress-strain state: Abstract of PhD dissertation. — Bukhara, 2025. — 116 p.

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