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PROBLEM STATEMENT AND METHOD FOR DERIVING THE DISPERSION EQUATION FOR WAVE PROPAGATION IN A THREE-LAYER VISCOELASTIC PLATE WITH A FILLER

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Abstract
This article presents the mathematical formulation of the problem of harmonic wave propagation in a three-layer viscoelastic plate with a filler, the solution methodology, and the method for obtaining the dispersion equation. Various contact conditions between the layers (rigid bonding, sliding, elastic connection) are formalized, the Lame equations are reduced, via Green–Lamb potentials, to a system of first-order ordinary differential equations, and a dispersion relation accounting for the viscoelastic properties of the material is derived on the basis of the Rzhanitsyn–Koltunov hereditary kernel. The results obtained serve as the basis for the numerical analysis of wave propagation in three-layer rods and plates presented in subsequent sections.

Keywords: Dispersion equation, viscoelasticity, three-layer plate, Rzhanitsyn-Koltunov kernel, Lame equation, boundary condition, global damping coefficient.

Introduction

Annotatsiya
Ushbu maqolada to'ldiruvchili uch qatlamli qovushqoq-elastik plastinkada garmonik to'lqin tarqalishi masalasining matematik qo'yilishi, uni yechish uslubiyoti va dispersion tenglamani olish usuli bayon etilgan. Qatlamlar orasidagi turli kontakt shartlari (qattiq mahkamlanganlik, sirpanuvchanlik, elastik bog'lanish) rasmiylashtirilgan, Lame tenglamalari Grin–Lemb potensiallari orqali birinchi tartibli oddiy differensial tenglamalar sistemasiga keltirilgan, hamda Rjanitsin–Koltunov irsiy yadrosi asosida qovushqoq-elastiklik xususiyatini hisobga oluvchi dispersion munosabat chiqarilgan. Olingan natijalar keyingi bo'limlarda uch qatlamli sterjen va plastinkalarda to'lqin tarqalishining sonli tahlili uchun asos bo'lib xizmat qiladi.

Kalit so'zlar: dispersion tenglama, qovushqoq-elastiklik, uch qatlamli plastinka, Rjanitsin-Koltunov yadrosi, Lame tenglamasi, chegaraviy shart, global so'nish koeffitsiyenti.

Аннотация
В данной статье изложена математическая постановка задачи распространения гармонических волн в трёхслойной вязкоупругой пластине с заполнителем, методика её

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решения и способ получения дисперсионного уравнения. Формализованы различные условия контакта между слоями (жёсткое соединение, скольжение, упругая связь), уравнения Ламе с помощью потенциалов Грина–Лэмба сведены к системе обыкновенных дифференциальных уравнений первого порядка, а также на основе наследственного ядра Ржаницына–Колтунова выведено дисперсионное соотношение, учитывающее вязкоупругие свойства материала. Полученные результаты служат основой для численного анализа распространения волн в трёхслойных стержнях и пластинах, представленного в последующих разделах.

Ключевые слова: дисперсионное уравнение, вязкоупругость, трёхслойная пластина, ядро Ржаницына-Колтунова, уравнение Ламе, граничное условие, коэффициент глобального затухания.

INTRODUCTION

The first and decisive stage in studying wave propagation and vibration processes in three-layer plates with a filler is the correct mathematical formulation of the problem and the development of a consistent methodology for solving it. In the late twentieth century, the scientific schools of M.A. Koltunov and I.Ye. Troyanovsky laid the foundation of the "quasi-static deformation theory" for dissipatively homogeneous and inhomogeneous mechanical systems, a theory later generalized in the works of V.P. Mayboroda, M.M. Mirsaidov, I.I. Safarov and other researchers. It follows from these studies that if a mechanical system consists of a combination of viscoelastic and elastic elements, the energy dissipation properties of the system differ fundamentally from those of a dissipatively homogeneous system.

Given the rapid development of the road and airfield construction industry, using materials with dissipatively inhomogeneous properties is one of the important reserves for increasing the strength of pavements. It is therefore of pressing scientific and practical importance to formulate the problem of wave propagation in long three-layer filled dissipatively homogeneous and inhomogeneous plates, and to develop a methodology and algorithm for solving it. The purpose of this article is to present, in a consistent manner, the complete mathematical formulation of this problem, the method of seeking a solution, and the process of deriving the dispersion equation.

1. Problem statement

Let wave propagation in long plate-like and cylindrical viscoelastic (dissipatively homogeneous and inhomogeneous) bodies be given in a spatial coordinate system. The dynamic processes arising from the propagation of harmonic waves in these bodies, as well as the dynamic stress-strain state (DSS) under forced harmonic forces, are studied. The equation of motion of an elementary parallelepiped mentally isolated from the medium is written in tensor form as:

$$(\tilde{\lambda}_k + 2\tilde{\mu}_k)\cdot \mathrm{graddiv}\, \vec{u}_k - \tilde{\mu}_k\cdot \mathrm{rotrot}\, \vec{u}_k + \vec{b}_k = \rho_k\cdot \partial^2\vec{u}_k/\partial t^2 \qquad (1)$$

where $\tilde{\lambda}_k$ and $\tilde{\mu}_k$ ($k=1,2,3$ — layer number) are the operator elastic moduli, defined as

$$\tilde{\lambda}_k f(t) = \lambda_{0k}\left[f(t) - \int_{-\infty}^{t} R_\lambda^{(k)}(t-\tau)f(\tau)d\tau\right], \quad \tilde{\mu}_k f(t) = \mu_{0k}\left[f(t) - \int_{-\infty}^{t} R_\mu^{(k)}(t-\tau)f(\tau)d\tau\right] \qquad (2)$$

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where $\vec{b}k$ is the body force ($bk=0$), ρk is the material density, $R\mu^{(k)}$ and $R\lambda^{(k)}$ are the relaxation kernels, $\lambda 0k$ and $\mu 0k$ are the instantaneous elastic moduli, and $\vec{u}k$ is the displacement vector. For small linear vibrations, taking $f(t) = \psi(t)e^{(-i\omega Rt)}$ and applying the freezing method, equation (1) is reduced to

$$(\bar{\lambda}k + 2\bar{\mu}k)\cdot \mathrm{graddiv}\,\vec{u}k - \bar{\mu}k\cdot \mathrm{rotrot}\,\vec{u}k = \rho k\cdot \partial^2\vec{u}k/\partial t^2 \quad (3)$$

For a three-layer filled structure, one of the following boundary (contact) conditions may be imposed:

— rigid bonding condition: at $z=hk$, $\sigma zzk=\sigma zz(k+1)$, $\sigma yzk=\sigma yz(k+1)$, $\sigma xyk=\sigma xy(k+1)$, $uk=uk+1$, $\vartheta k=\vartheta k+1$, $wk=wk+1$;

— slip condition (no resistance between layers): $\sigma zzk=\sigma zz(k+1)$, $\sigma zyk=\sigma zxk=0$, $wk=wk+1$;

— elastic bonding (accounting for resistance): $\sigma zyk=\sigma zxk=kz\cdot\sigma zzk$, $wk=wk+1$;

— free-surface condition (stress-free) at the plate's outer surface: $\sigma rrN=0$, $\sigma r\theta N=0$, $\sigma rzN=0$.

If a massless element is placed between the layers, the contact condition is written as

$$\sigma zzk = \bar{k}z\cdot(uzk - uz(k+1)),\quad \sigma zxk = \bar{k}x\cdot(uxk - ux(k+1)) \quad (4)$$

For the outer layers of the plate, a stress-free condition is imposed, while as the radial coordinate tends to infinity, the Sommerfeld radiation condition is imposed on the longitudinal and transverse wave potentials [46, 49]. As initial conditions, all displacements and their velocities are taken to be zero at $t=0$.

For the case where the structure is in contact with the surrounding medium through massless deformable elements (Winkler foundation), the external action is accounted for as

$$q1z^k = -\tilde{k}z^k\cdot w^k,\quad \tilde{k}z^k[f(t)] = kz0^k[f(t) - \int_0^t Rk(t-\tau)f(\tau)d\tau] \quad (5)$$

where $kz0^k$ is the instantaneous bed coefficient. The vector equations (3) are reduced to the Lame equations in a plane:

$$\bar{\mu}k\cdot(\partial^2uk/\partial x^2 + \partial^2uk/\partial z^2) + (\bar{\lambda}k+\bar{\mu}k)\cdot\partial/\partial x(\partial uk/\partial x + \partial\vartheta k/\partial z) - \rho k\cdot\partial^2uk/\partial t^2 = 0 \quad (6)$$

For plane strain, the continuity conditions for the stress tensor and displacement vector at a rigidly bonded boundary are written as $\sigma_{33}^{(1)}=\sigma_{33}^{(2)}$, $\sigma_{13}^{(1)}=\sigma_{13}^{(2)}$, $u_1=u_2$, $\vartheta_1=\vartheta_2$ (7); at a frictionless boundary as $\sigma_{33}^{(1)}=\sigma_{33}^{(2)}$, $\sigma_{13}^{(1)}=\sigma_{13}^{(2)}=0$, $\vartheta_1=\vartheta_2$ (8); and at a free surface as $\sigma_{33}^{(1)}=0$, $\sigma_{13}^{(1)}=0$ (9). The problem is solved in dimensionless parameters ($x=h\pi^{-1}\tilde{x}$, $y=h\pi^{-1}\tilde{y}$, $z=h\pi^{-1}\tilde{z}$, $\omega=h\pi^{-1}\omega cs01^{-1}$), where $cs01$ is the instantaneous transverse wave velocity in the first layer.

2. Solution methodology

The Green–Lamb transformation is used to solve the stated problem:

$$\vec{u}k = \mathrm{grad}\varphi k + \mathrm{rot}\,\vec{\psi}k,\quad \mathrm{div}\,\vec{\psi}k = 0 \quad (10)$$

In Cartesian coordinates this expression is written as

$$uxk = \partial\varphi k/\partial x + \partial\psi zk/\partial y - \partial\psi yk/\partial z,\quad uyk = \partial\varphi k/\partial y + \partial\psi xk/\partial z - \partial\psi zk/\partial x,\quad uzk = \partial\varphi k/\partial z + \partial\psi yk/\partial x - \partial\psi xk/\partial y \quad (11)$$

where φk is the longitudinal wave potential and $\vec{\psi}k(\psi xk,\psi yk,\psi zk)$ is the transverse wave potential. Substituting (10) into (3) yields the complex-coefficient wave equations

$$(1-i\Gamma\lambda\mu k)\cdot\nabla^2\varphi k = (1/Cp0k^2)\cdot\partial^2\varphi k/\partial t^2,\quad (1-i\Gamma\mu k)\cdot\nabla^2\vec{\psi}k = (1/Cs0k^2)\cdot\partial^2\vec{\psi}k/\partial t^2 \quad (12)$$

where $Cp0k^2 = (\lambda 0k+2\mu 0k)/\rho k$ and $Cs0k^2 = \mu 0k/\rho k$ are the instantaneous longitudinal and transverse wave velocities, and $\Gamma\mu k$ and $\Gamma\lambda\mu k$ are complex parameters expressed through the

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cosine and sine Fourier images of the relaxation kernel. The solution, decaying as $x\to\infty$, is sought in the form

$$\varphi_k = C_{1k}\cdot e^{-r_{1k}\cdot x}\cdot e^{i(\omega t-(\gamma y+sz))},\quad \vec{\psi}_k = \vec{C}_k\cdot e^{-r_{2k}\cdot x}\cdot e^{i(\omega t-(\gamma y+sz))} \qquad (13)$$

where γ and s are the wavenumbers along the y and z axes, and C_{1k} and $\vec{C}_k=(C_{3k},C_{4k},C_{2k})^T$ are integration constants (four for each layer: $C_{1k},C_{2k},C_{3k},C_{4k}$). Substituting (13) into the wave equation yields the relations

$$r_{1k} = \sqrt{\gamma^2+s^2-\bar{k}^2\omega^2},\quad r_{2k} = \sqrt{\gamma^2+s^2-\omega^2},\quad k = \sqrt{(1-2\nu)/(2(1-\nu))} \qquad (14)$$

The wave decay condition as $x\to\infty$ is $\gamma^2+s^2>\omega^2$. Substituting the resulting solutions (13)–(14) into (11) fully determines the displacement and stress fields of the three-layer plate and filler.
For plane strain, the solution is sought in the form $\sigma_{ij}^{(k)} = \sigma_{ij}^k(z)\cdot \exp(i\gamma x-i\omega t)$, $u_k = U_k(z)\cdot \exp(i\gamma x-i\omega t)$, $\vartheta_k = V_k(z)\cdot \exp(i\gamma x-i\omega t)$ (15), where $\sigma_{ij}^k(z)$, $U_k(z)$, $V_k(z)$ are amplitude vector-functions, γ is the wavenumber, and $S=S_R+iS_I$ is the phase velocity. Substituting this solution into the equation of motion yields a system of first-order ordinary differential equations

$$d\sigma_{13k}/dz = E_{4k}\cdot\sigma_{33k} - E_{5k}\cdot U_k,\quad d\sigma_{33k}/dz = \delta_k\cdot\sigma_{31k} - \Omega_k^2\cdot V_k,\quad dU_k/dz = E_{1k}\cdot\sigma_{33k} - E_{2k}\cdot U_k,\quad dV_k/dz = E_{3k}\cdot\sigma_{13k} + \delta_k\cdot V_k \qquad (16)$$

where the coefficients $E_{1k},\dots,E_{5k}$, δ_k and Ω_k are determined by the layer parameters (E_k, ν_k, and the quantities E_{kF}, ν_{kF} expressed through Rabotnov's fractional-exponential function) and the frequency ω. At the interface ($z=h_k$), boundary conditions similar to (7)–(9) are imposed, formulating a spectral boundary-value problem with respect to the parameter ω. This spectral problem is solved in complex arithmetic through successive application of the orthogonal sweep method and Muller's method, and in some more complex cases through a methodology and algorithm based on finite differences, the Laplace method, and the Gauss method. When an external harmonic dynamic force is applied to a half-space, the problem reduces to a system of inhomogeneous algebraic equations, solved by the Gauss method.
Finding the arbitrary constants (integration constants) leads to a system of homogeneous linear algebraic equations expressing the boundary and contact conditions. For this system to have a nontrivial solution, it is necessary and sufficient that its main determinant equal zero — this condition yields the dispersion relation.

3. Deriving the dispersion equation

The dispersion equation constructed for a deformable layer is generally written as

$$\det[R(k,\omega)] = 0 \qquad (17)$$

Here the equation expressing the relationship between the wavenumber k and the frequency ω may have n complex roots: $k_j = k_j(\omega) = \mathrm{Re}\,k_j(\omega) + i\cdot \mathrm{Im}\,k_j(\omega)$, $j=1\dots n$; each root is called a wave mode. If $k=k_1+ik_2$, then $u_z(x,t)=A(x,t)\cdot e^{-k_2 x}\cdot e^{i(k_1 x+\omega t)}$, where: for $k_2>0$ the wave amplitude decays exponentially (dissipation); for $k_2<0$ the amplitude grows without bound (an unstable process); for $k_2=0$ the wave propagates with undamped amplitude; and for $k_1=0$ a standing wave results. The dispersion equation has the same form as the corresponding equation for an elastic layer [23, 27, 35], but unlike the latter, in the hereditary-elastic case its coefficients are complex-variable functions and it does not have complex-conjugate roots, so that the symmetry of the frequency spectrum is broken and the branches of the hereditary-elastic spectrum split.

Seeking the solution in form (15) yields a system of second-order ordinary differential equations

$\bar{\mu}_k\cdot(U_k''-k^2U_k) + (\bar{\lambda}_k+\bar{\mu}_k)\cdot i\gamma\cdot(i\gamma U_k+V_k') + \rho_k\cdot\gamma^2C^2U_k = 0$, $\bar{\mu}_k\cdot(V_k''-\gamma^2V_k) + (\bar{\lambda}_k+\bar{\mu}_k)\cdot(i\gamma U_k'+V_k'') + \rho_k\cdot k^2C^2V_k = 0$ (18)

The condition for the resulting homogeneous linear algebraic system of equations for the particular solutions to have a nontrivial solution is that the main determinant equal zero:

$| \ \bar{C}_{Tk}^2(r_k^2-\gamma^2)-(\bar{C}_{Lk}^2-\bar{C}_{Tk}^2)\gamma^2+\bar{C}^2\gamma^2 \quad i(\bar{C}_{Lk}^2-\bar{C}_{Tk}^2)\gamma r_k \ ; \ i(\bar{C}_{Lk}^2-\bar{C}_{Tk}^2)\gamma r_k \quad \bar{C}_{Lk}^2(r_k^2-\gamma^2)+(\bar{C}_{Lk}^2-\bar{C}_{Tk}^2)r_k^2+C^2\gamma^2 \ | = 0$ (19)

where $\bar{C}_{Lk}^2 = (\bar{\lambda}_k+2\bar{\mu}_k)/\rho_k$ and $\bar{C}_{Tk}^2 = \bar{\mu}_k/\rho_k$ are the compressional and shear wave velocities, respectively (coinciding with the corresponding values in an elastic medium when $R(t)=0$). For each value of k, equation (19) has four roots: $(r_k)_{1,2} = \pm\gamma\sqrt{(1-C^2/\bar{C}_{Lk}^2)}$, $(r_k)_{3,4} = \pm\gamma\sqrt{(1-C^2/\bar{C}_{Tk}^2)}$. Separating the states symmetric and antisymmetric with respect to the normal coordinate, and applying standard substitutions, yields two dispersion equations:

$\gamma^4\cdot\cos(a)\cdot\sin(b)/b - a^2k^2\cdot\cos(b) = 0$ (20, symmetric case)

$\gamma^4\cdot\cos(b)\cdot\sin(a)/a - b^2\bar{k}^2\cdot\cos(a)\cdot\sin(b)/b = 0$ (21, antisymmetric case)

where $a^2 = \bar{k}_*^2 - k_F^2\Omega_*^2$, $b^2 = \bar{k}_*^2 - \Omega_*^2$, $k_F^2 = (1-2\nu_{F*})/(2-2\nu_{F*})$, $k_*=h_1k$, $E_{*F} = 1 - m_*/(\beta_*+\sqrt{(i\omega)})$, $\nu_{*F} = \nu_1 + [(1-2\nu_1)/2]\cdot m_*/(\beta_*+\sqrt{(i\omega_*)})$. Formally these equations have the same form as the dispersion equation of the corresponding elastic layer [92], but in the hereditary-elastic case the left-hand side consists of complex-variable coefficients. As an example, the problem of free wave propagation in a viscoelastic layer lying on a half-space [92, 94] is considered under the following boundary conditions: at $z=h$, $\sigma_{zz1}=0$, $\sigma_{zx1}=0$; at $z=0$, $\sigma_{zz1}=\sigma_{zz2}$, $\sigma_{zx1}=\sigma_{zx2}$, $u_1=u_2$, $\vartheta_1=\vartheta_2$; and as $z\to-\infty$, $u_2\to0$, $\vartheta_2\to0$.

The three-parameter Rzhanitsyn–Koltunov kernel is used to express the relaxation kernel of the viscoelastic material:

$$R(t) = A\cdot e^{(-\beta t)}/t^{(1-\alpha)} \qquad (22)$$

where A, α, β are material parameters, taken to have the following values: $A=0.048$; $\beta=0.05$; $\alpha=0.1$ [61]. The roots of the resulting dispersion equation are found using Muller's method, with the Gauss method (with pivoting) applied at each iteration; in this case, direct expansion of the determinant is not required. The improper integral in the equation is replaced by the complex relation $\int_{-\infty}^{t} R(t-\tau)\varphi(\tau)d\tau = [i\Gamma_s(\Omega)+\Gamma_c(\Omega)]\cdot\varphi(t)$ (23), where Γ_c and Γ_s are the cosine and sine Fourier transforms of the relaxation kernel.

At a fixed value of the damping coefficient, the minimum value of the filler's stiffness is of significant importance: $\delta\omega = \min(-\omega I_k)$, $k=1,2,...,K$ (24). Here $\delta\omega$ is the system's global damping coefficient. For a dissipatively homogeneous system, the coefficient $\delta\omega$ is fully determined by the imaginary part of the first complex frequency; for a dissipatively inhomogeneous system, this role is played by the imaginary part of the first or second frequency (depending on their values), and the "switching of roles" occurs at the stiffness value at which the real parts of the first and second frequencies are closest to each other. This concept of the global damping coefficient (GDC), introduced by I.Ye. Troyanovsky and I.I. Safarov, together

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with the related "Troyanovsky–Safarov effect," is the primary tool for characterizing the energy dissipation properties of dissipatively inhomogeneous three-layer plate systems.

CONCLUSION

This article presented the complete mathematical formulation of the problem of wave propagation and vibration in a three-layer viscoelastic plate with a filler: the equations of motion were written in tensor form, and all possible contact conditions between the layers (rigid bonding, slip, elastic connection, free surface) were formalized. Using Green–Lamb potentials, the problem was reduced to complex-coefficient wave equations and then to a system of first-order ordinary differential equations, solved via the orthogonal sweep method and Muller's method.

Accounting for viscoelastic properties through the Rzhanitsyn–Koltunov hereditary kernel (A=0.048; β=0.05; α=0.1), two independent dispersion equations were obtained for the states symmetric and antisymmetric with respect to the normal coordinate. The fundamental difference between these equations and the dispersion equation of the corresponding elastic layer is that their coefficients are complex-variable functions, so that complex-conjugate roots do not exist and the symmetry of the frequency spectrum is broken.

The developed methodology, together with the concept of the I.Ye. Troyanovsky – I.I. Safarov global damping coefficient, serves as the basis for the subsequent articles — obtaining exact numerical results for wave propagation in three-layer rods and plates, and analyzing the dynamic stress-strain state under harmonic loading.

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