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PROPERTIES OF NONLINEAR OSCILLATIONS OF DISSIPATIVE MECHANICAL SYSTEMS WITH A LIMITED NUMBER OF DEGREES OF FREEDOM

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Abstract

This article studies nonlinear oscillation processes of dissipative mechanical systems with a limited number of degrees of freedom. The properties of dissipative forces, their influence on nonlinear oscillations, the stability of oscillations, the formation of limit cycles, subharmonic and superharmonic resonance phenomena are analyzed. Also, the main mechanisms of nonlinear oscillations are considered using spatial portraits, Lyapunov stability criteria, parametric oscillations, Duffing, Van der Pol and Rayleigh-type models. The energy balance of systems, the effect of the degree of dissipation on the time evolution of the oscillation amplitude are also covered from a mathematical point of view.

Keywords: Nonlinear vibrations, dissipative system, degree of freedom, stability, spatial portrait, resonance, limit cycle, Lyapunov function, Duffing model, Van der Pol vibrations.

Introduction

Vibration processes of mechanical systems are a fundamental component of many natural and technical systems. Real mechanical systems are often not ideal and have energy losses, i.e., dissipation. In addition, in many processes, forces obey not linear laws, but nonlinear laws. Therefore, the use of nonlinear and dissipative models in the study of vibrations of real systems is important. Systems with a limited number of degrees of freedom constitute a separate section of vibration theory. Such systems include mechanical oscillators, robot manipulator joints, mechanical structures connected by elastic elements, and actuators of automatic control systems. The behavior of these systems is often described not by simple linear models, but by complex nonlinear equations. The purpose of this article is to analyze dissipative and nonlinear vibrations from a mathematical and physical point of view, to study important mechanisms such as their stability properties, resonance processes, and the formation of limit cycles.

Main Part

General description of dissipative systems.

A dissipative system is a system in which energy decays over time, the amplitude of oscillations decreases even in the absence of external influences. Dissipation often arises from the following sources:

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Internal resistance (internal friction of the material).
External friction (air resistance, mechanical friction).
Energy consumption of active control systems.
The classical model for a system with a simple one degree of freedom:

$$m\ddot{x} + c\dot{x} + kx = 0$$

This is a linear model, and in real systems the following nonlinear additions are often introduced:

$$m\ddot{x} + c\dot{x} + kx + \alpha x^3 = 0$$

This is a Duffing-type nonlinear oscillator.

Reasons for the formation of nonlinear vibrations.

Nonlinear vibrations appear in the following cases:
- Nonlinearity of the material
- Large deformations
- Geometric nonlinearity of contact forces
- Various elastic modes of structures
- Aerodynamic effects (flatter, buffeting)
- Backlash in mechanical connections

As a result of these reasons, the vibration equations become more complicated and their solutions also differ from those in linear systems.

The interaction of dissipation and nonlinearity.

The presence of dissipation strongly affects the amplitude of the oscillations of the system. In linear systems, energy loss damps the oscillations, while in nonlinear systems:
- The appearance of a limit cycle
- Self-oscillation (self-oscillation)
- Complex resonant responses

Hysteresis phenomena are observed.

For example, in a Van der Pol oscillator:

$$\ddot{x}-\mu(1-x^2)\dot{x} + x = 0$$

This equation describes a system with dissipative but nonlinear self-oscillations.
The interaction of dissipation and nonlinearity.
The presence of dissipation strongly affects the amplitude of the oscillations of the system. In linear systems, energy loss damps the oscillations, while in nonlinear systems:
- The appearance of a limit cycle
- Autooscillation (self-oscillation)
- Complex resonant responses

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For example, in a Van der Pol oscillator:

$$\ddot{x}-\mu(1-x^2)\dot{x} + x = 0$$

This equation describes a system with dissipative but nonlinear autooscillations.
Spatial portraits and stability

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Spatial portraits are important in the analysis of nonlinear systems. The following are observed in the spatial portrait:

1. Stable focus
2. Unstable focus
3. Limit cycles
4. Multi-cycle regimes
5. Linear convergence
6. Strange attractors (in some systems)

The Lyapunov function method allows you to assess the stability of a dissipative system.
If for some function: $\dot{V} < 0$, the system is considered asymptotically stable.

Resonance phenomena

Resonance manifests itself in nonlinear systems as follows: Superharmonic resonance (in frequency dividers), Subharmonic resonance (in multipliers)

Combination resonance

The frequency-amplitude characteristic of nonlinear systems is usually curved and has hysteresis. For example, in the Duffing oscillator, the phenomenon of amplitude jumps is observed as a result of the nonlinearity of the stiffness.

Formation of limit cycles

A limit cycle is a stable periodic trajectory of oscillations. In nonlinear systems with dissipation, there may be:

- A single stable limit cycle
- Multiple limit cycles

Complex structures arising from internal resonances, the Van der Pol equation being a classic example, in which a limit cycle appears regardless of any initial conditions.

Systems with multiple degrees of freedom

Dissipative systems with multiple degrees of freedom are described by the following equation:

$M\ddot{x}+C\dot{x}+Kx+f(x,\dot{x}) = Q(t)$

where:

– Mass matrix
– Dissipation matrix
– Stiffness matrix
– Nonlinear forces
– External force
– External force

The following properties appear in multiple degrees of freedom:

- Internal resonances (1:2 oscillations
- Mode coupling (interference of modes)
- Amplitude modulation

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• Complex quasiperiodic oscillations

Energy analysis.

The energy balance in dissipative systems is:

\frac{dE}{dt} = -D(x,\dot{x}) + P(t) where:

– Dissipation function

– External work

In this article, we consider the linear equations of dissipative mechanical systems with a finite number of degrees of freedom. nonlinear oscillations were analyzed extensively. The role of dissipation, various sources of nonlinearity, resonance phenomena, spatial portraits, limit cycles, and stability issues were considered. Nonlinear dissipative systems have a much richer dynamic behavior than linear systems, and their analysis is one of the main directions of modern theoretical and applied mechanics. Systems of this type are found in many technical devices, and their correct modeling and control are of great practical importance.

References

1. Nayfeh A.H., Mook D.T. Nonlinear Oscillations. Wiley-Interscience.
2. Verhulst F. Nonlinear Differential Equations and Dynamical Systems. Springer.
3. Stoker J.J. Nonlinear Vibrations in Mechanical and Electrical Systems. Wiley.
4. Khalil H. Nonlinear Systems. Prentice Hall.
5. Bogoliubov N., Mitropolsky Y. Asymptotic Methods in the Theory of Nonlinear Oscillations.
6. Kovacic I., Brennan M. The Duffing Equation: Nonlinear Oscillators and their Behaviour.

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