



CALCULATION METHOD FOR THE DIFFUSION MIXING DYNAMICS OF COLLECTOR WATERS IN IRRIGATION CANALS (CASE STUDY OF THE KHOREZM REGION)

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Abstract

This paper is devoted to improving a hydraulic model describing the change in salt concentration in canal sections where collector water is discharged from the main collectors of the Khorezm region into irrigation canals via pumping stations. The diffusion equation was approximated in finite-difference form using the Samarskii method, and its numerical solution was obtained by the sweep (progonka) method. The numerical results of the developed model were compared with field data using Student's method, and the average discrepancy was found to be 6.1%. The results obtained provide a basis for developing practical recommendations for managing the safe discharge of collector-drainage water into irrigation canals.

Keywords: Collector water, irrigation canals, salt concentration, diffusion mixing, hydraulic model, diffusion equation, numerical experiment, finite difference method.

Introduction

At present, considerable attention is being given to the scientific analysis of the quantitative and qualitative indicators of collector-drainage water in large main collectors, to the improvement of hydraulic engineering solutions that enable the safe discharge of part of this water into irrigation canals, and to the development of scientifically grounded mechanisms for mixing canal and collector water for use in irrigating agricultural crops. In particular, the relevance of this study is determined by the need to improve a diffusion-convective model capable of accurately predicting the dynamics of salt concentration when collector water is discharged into irrigation canals.

In particular, water is drawn from the Atashev, Tashsaqa Ung Qirgoq, Divankul, and Daryoliq main collectors of the Khorezm region through 56 pumping stations and discharged into more than 30 irrigation canals in the Khonqa, Urganch, Yangibozor, and Gurlan districts [2,4,5]. The mineralization of water in these collectors averages 2.1–2.6 g/l (Table 1).

Table 1 Volume of water discharged from the Daryoliq collector in 2004–2024

Year	Indicator	Unit	Annual average
2004	Flow rate	m ³ /s	23,25
	Runoff	mln.m ³	734,86
	Mineral.	g/l	2,47
2005	Flow rate	m ³ /s	31,16
	Runoff	mln.m ³	992,37
	Mineral.	g/l	2,29
	Runoff	mln.m ³	507,53
	Mineral.	g/l	2,23
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2023	Flow rate	m ³ /s	16,74
	Runoff	mln.m ³	531,68
	Mineral.	g/l	2,31
2024	Flow rate	m ³ /s	19,90
	Runoff	mln.m ³	631,27
	Mineral.	g/l	2,28

In the above-mentioned irrigation canals, this value varies within the range of 1.2–1.4 g/l. Within this study, the change in salt concentration in canal water was experimentally and theoretically investigated at three discharge sites: PK 182+70 of the “Tashsaqa Ung Qirgoq” collector, which supplies the “L-7-7” canal (PK 14+50) in the “Amudaryo” area of Khonqa district; PK 61+00 of the “Atashev” collector, which supplies the “Durmanyab” canal (PK 155+00) in the “Durman” area of Urganch district; and PK 353+00 of the “Divankul” collector, which supplies the “Dukonboy” canal (PK 120+00) in the “Guliston” area of Gurlan district — all sites where pumping stations discharge collector water into these canals. The average values of laboratory analyses of water mineralization and ion concentration at the sites where collector water is discharged into irrigation canals are given in Table 2 [4,5].

Table 2

No.	Irrigation canal picket	Ion concentration			Concentration, %
		Nitrates	Chlorides	Sulfates	
		g/l	g/l	g/l	
1.	“L-7-7” canal, PK 14+50	1,42	1,54	1,54	0,18
2.	“Durmanyab” canal, PK 155+00	1,54	1,65	1,56	0,22
3.	“Dukonboy” canal, PK 120+00	1,61	1,57	1,61	0,25

In line with the research objective, we improve the hydraulic model describing the change in salt concentration in canal sections where collector water is discharged. For this purpose, we use the diffusion equation from diffusion theory [1,2,3]:

$$\frac{\partial c}{\partial t} + \text{div} \vec{j} + \sigma c = \text{div} \vec{l} \quad (1)$$

where: $\vec{j} = cV_r \vec{e}_r + cV_{\text{water}} \vec{e}_\theta - (cV_{\text{water}} + D \frac{\partial c}{\partial x}) \vec{e}_x$, $\vec{l} = c_n \vec{V}_{\text{water}} \vec{e}_q$

D – is the coefficient of turbulent diffusion; V_0 – is the average flow velocity in the canal; $\text{div} \vec{l}$ – is the intensity of the diffusion flux, $\frac{1}{s}$. Equation (1) can be written in the following form:

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} = D \frac{\partial^2 c}{\partial x^2} + f(x, t) \quad (2)$$

where: c – is the ion concentration in canal water, %;

$f(x, t) = \frac{c_{col} \int_0^T Q_{col} dt + c_{can} \int_0^T Q_{can} dt}{(c_{col} + c_{can}) [\int_0^T Q_{col} dt - \int_0^T Q_{can} dt]} \cdot \text{div} \vec{l}$ – is a function characterizing the intensity of diffusion mixing of collector and canal waters. (2) To carry out the numerical experiment for this equation, we use the method of A.A. Samarskii. Assuming $u = V_0 = \text{const} > 0$, we first perform the following approximation for the convective terms of equation (2):

$$\frac{\partial c}{\partial x} \approx \frac{c(x, \tau) - c(x-h, \tau)}{h} = \frac{c_k - c_{k-1}}{h}$$

c_{k-1} is expressed via a Taylor series in terms of c_k :

$$c_{k-1} = c_k - \frac{\partial c}{\partial x} h + \frac{\partial^2 c}{\partial x^2} \frac{h^2}{2} - \frac{\partial^3 c}{\partial x^3} \frac{h^3}{3!} + o(h^4),$$

$$\frac{\partial c}{\partial x} + o(h^2) = \frac{c_k - c_{k-1}}{h} + \frac{\partial^2 c}{\partial x^2} \frac{h^2}{2} \quad (3)$$

(3) — substituting this expression into (2), we obtain:

$$\frac{c_k^{n+1} - c_k^n}{\Delta \tau} + V \frac{c_k^{n+1} - c_{k-1}^{n+1}}{h} = D \frac{\partial^2 c}{\partial x^2} - \frac{Vh}{2} \frac{\partial^2 c}{\partial x^2} + f$$

We also write the second-order derivative and the inertial terms in finite-difference form at the time layer $n + 1$:

$$\frac{c_k^{n+1} - c_k^n}{\Delta \tau} + V \frac{c_k^{n+1} - c_{k-1}^{n+1}}{h} = (D - \frac{Vh}{2}) \frac{c_{k+1}^{n+1} - 2c_k^{n+1} + c_{k-1}^{n+1}}{h^2} + f_k^n \quad (4)$$

$\alpha = 0$ we expand the following Taylor series:

$$\frac{1}{1+\alpha} = 1 - \alpha + \alpha^2 - \alpha^3 + \dots \quad (5)$$

If $\alpha = 1$, then from series (5) we obtain:

$$\frac{1}{1+\alpha} \approx 1 - \alpha \quad (6)$$

$\frac{Vh}{2}$ for small values of:

$$1 - \frac{Vh}{2} = \frac{1}{1 + \frac{Vh}{2}} \quad (7)$$

Then (4) can be expressed as follows:

$$\frac{c_k^{n+1} - c_k^n}{\Delta \tau} + V \frac{c_k^{n+1} - c_{k-1}^{n+1}}{h} = \frac{c_{k+1}^{n+1} - 2c_k^{n+1} + c_{k-1}^{n+1}}{(D + \frac{Vh}{2})h^2} + f_k^n$$

or

$$\frac{\Delta\tau}{(D+\frac{Vh}{2})h^2} c_{k+1}^{n+1} + [-1 - 2\frac{\Delta\tau}{(D+\frac{Vh}{2})h^2} - V\frac{\Delta\tau}{h}] c_k^{n+1} + \left[\frac{\Delta\tau}{(D+\frac{Vh}{2})h^2} c_{k+1}^{n+1} + V\frac{\Delta\tau}{h} \right] c_{k-1}^{n+1} = -c_k^n - \Delta\tau f_k^n + o(\Delta\tau + h^2) \quad (8)$$

Thus, equation (2) is approximated with an error of order $o(\Delta\tau + h^2)$ by the finite-difference equation (8).

From the discrete equation (8), we obtain the following solution:

$$c_k^{n+1} = \left\{ \frac{\Delta\tau}{(D+\frac{V_k h}{2})h^2} - \frac{V_k - |V_k| \Delta\tau}{2h} \right\} c_{k+1}^n + \left\{ 1 - \frac{2\Delta\tau}{(D+\frac{V_k h}{2})h^2} - \frac{|V_k| \Delta\tau}{2h} \right\} c_k^n + \left\{ \frac{\Delta\tau}{(D+\frac{|V_k| h}{2})h^2} + \frac{V_k - |V_k| \Delta\tau}{2h} \right\} c_{k-1}^n + \Delta\tau f_k^n + o(\Delta\tau + h^2) \quad (9)$$

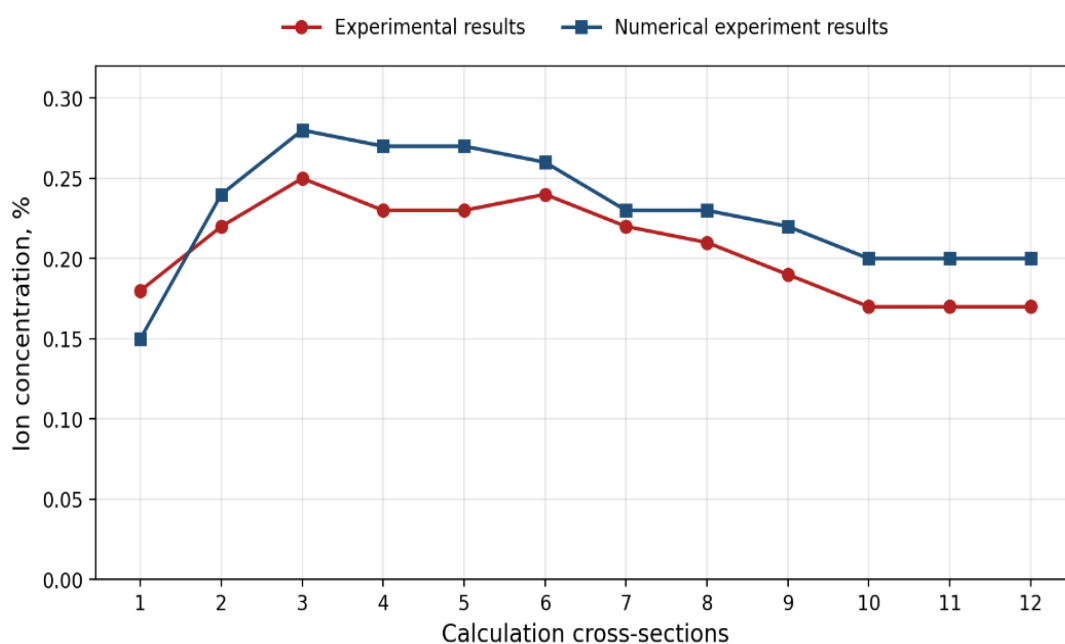


Figure 1. Dynamics of ion concentration in water resulting from diffusion mixing of collector and irrigation canal waters

As a result, (2) was reduced to its discrete-form solution. Based on the experimental studies carried out at the research site, the numerical experiment for the discrete equation (9) was carried out using the sweep (progonka) method.

(9) A comparison graph of the numerical experiment results for the discrete equation and the results of the field experimental study is presented in Figure 1. The comparison error was determined using Student's method and averaged 6.1%.

**References:**

1. Karasev I.F. Diffusion-convective model of turbulence and hydraulic characteristics of channel flows. *Meteorology and Hydrology*, Moscow: Gidrometeoizdat, 2013, No. 9, pp. 69–78. (in Russian)
2. Makhmudov I.E., Mamutov R.A. A semi-empirical expression for determining the hydraulic resistance coefficient in open horizontal collector-drainage networks. *Agro Ilm, Scientific-Practical Journal*, 2025, No. 9, pp. 166–167. (in Uzbek)
3. Lyakhter V.M., Militeev A.N., Shkolnikov S.Ya. Numerical modeling of turbulent open flows. *Water Resources*, 1997, No. 1, pp. 35–41. (in Russian)
4. Mamutov R.A., Murodov N.K., Ulugbekov B.B. Technologies for the reuse of collector water and their advantages (case study of Khorezm region). *Agro Ilm, Scientific-Practical Journal*, 2025, Special Issue No. 4, pp. 9–11. (in Uzbek)
5. R.A. Mamutov “Calculation method for diffusion mixing dynamics of collector waters in irrigation canals”. *Agricultural sciences and foodstuffs*. December 26, 2025. Amsterdam, The Netherlands. 169-172 pp.
6. Kolmogorov A.N. Local structure of turbulence in an incompressible viscous fluid at very large Reynolds numbers. 1981, vol. 30, No. 4, pp. 299–303. (in Russian)