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# SCHEME OF STUDYING FUNCTIONS FOR MONOTONICITY AND EXTREMUM

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**Abstract:**
The article discusses the scheme of studying a function for monotonicity and extremum

**Keywords**: scheme, study, function, monotonicity, extremum.

**Introduction**

Based on the formulated theorems of §§ 1–2, a function that is continuous on the interval $(a, b)$, except for perhaps a finite number of points, and has a finite number of critical points on this interval $f(x)$can be investigated for monotonicity and extremum according to the following scheme:

$1^0$. Find the domain of the function $f(x)$;

$2^0$. Find the derivative $f'(x)$;

$3^0$. Determine the critical points $f(x)$belonging to the region function definitions;

$4^0$. Using critical points, divide the domain of the function into non-intersecting intervals and determine the sign of the derivative on each interval $f'(x)$. If $f'(x) > 0$, then on the interval, the function $f(x)$increases, but if $f'(x) < 0$, then the function $f(x)$decreases;

$5^0$. Designate critical points on the numerical axis designate critical points on the numerical axis and, based on the signs of the derivatives defined in point 4 $f'(x)$, when passing through these points, determine whether these points are points of the extremum of the function $f(x)$:

$6^0$. Find the extrema of the function, if they exist.

***Note 1.*** To determine the sign of the derivative $f'(x)$on the interval in point 4, it is sufficient to determine the sign of the derivative $f'(x)$at any arbitrary point of this interval that is convenient for identifying the sign of the derivative.

***Remark 2.*** To determine the extremum point of the function $f(x)$in point 5, one can use higher-order derivatives of the given functions within the framework of Theorem 5–6.

*Example 1.* Investigate the monotonicity and extremum of a function.

$$f(x) = 1 + x^2 - \frac{x^4}{2},$$

specified on the number axis.

*Solution* : Let's conduct research on the points of the proposed scheme:

$1^0$. The given function is defined on the entire number axis.

$2^0$. We determine the derivative of the given function:



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$$f'(x) = 2x - 2x^3.$$

$3^0$. Let us determine the critical points of the given function, for which we first equate $f'(x)$to zero, i.e.

$$2x - 2x^3 = 0$$

we find that the last equality holds at the points . $x_1 = -1, x_2 = 0, x_3 = 1$.Therefore , the points $x_1, x_2, x_3$are critical points of the given function. Since the function has finite derivatives on the number axis, it has no other critical points.

$4^0$. Using the critical points found, we divide the domain of the function into non-intersecting intervals. $(-\infty, -1), (-1,0)$ , $(0,1), (1, +\infty)$.We select one point on each interval, in accordance with remark 1. Let these be the points$-2 \in (-\infty, -1), -\frac{1}{2} \in (-1,0), \frac{1}{2} \in (0,1), 2 \in (1, +\infty)$.

Let's determine the signs $f'(x)$at these points:

$$f'(-2) = 2(-2)(1-(-2)^2) = -4(-3) = 12 > 0\ ("+");$$

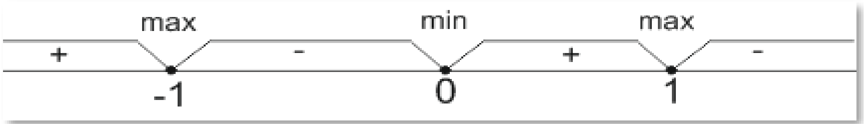
$$f'\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)\left(1-\left(-\frac{1}{2}\right)^2\right) = -1\left(1-\frac{1}{4}\right) = -\frac{3}{4} < 0\ ("-");$$

$$f'\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)\left(1-\left(\frac{1}{2}\right)^2\right) = \left(1-\frac{1}{4}\right) = \frac{3}{4} > 0\ ("+");$$

$$f'(2) = 2 \cdot 2(1-(2)^2) = 4(-3) = -12 < 0\ ("-").$$

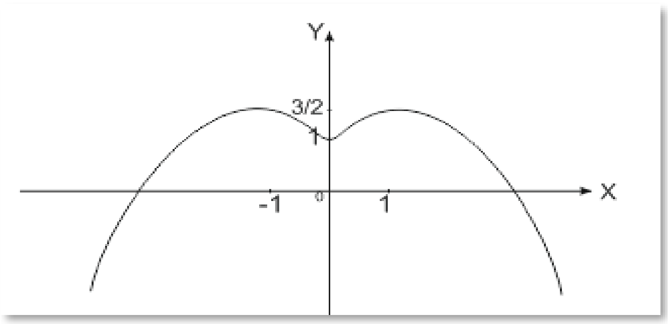
Therefore, on intervals $(-\infty, -1)$, $(0,1)$the function $f(x)$increases, on intervals $(-1,0)$, $(1, +\infty)$decreases.

$5^0$. Let us designate the critical points on the numerical axis using the derivative signs $f'(x)$when passing through these critical points, and determine whether they are extremum points:




$6^0$. Let's find the extrema of the function $f(x)$by calculating the values of the function $f(x)$at the maximum points - 1 and 1, as well as the minimum 0:

$$f(-1) = \frac{3}{2}, f(1) = \frac{3}{2}, f(0) = 1.$$




*Example 2.* Investigate the function for monotonicity and extremum.

$$f(x) = sin^2x - 5sinx + 6.$$

*Solution* : Let's conduct research on the points of the proposed scheme:

$1^0$. The given function is defined on the entire number axis.

$2^0$. We determine the derivative of the given function:

$$f'(x) = 2sinxcosx - 5cosx = cosx(2sinx - 5).$$

$3^0$. We determine the critical points of the given function by equating the derivative to zero



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and taking into account that $2sinx - 5 < 0$, then

$$cosx(2sinx - 5) = 0 \text{at} x_k = \frac{\pi}{2}(2k+1), k = 0; \pm1; \pm2; \pm3; \ldots.$$

$(x_k = \frac{\pi}{2}(2k+1), k = 0; \pm1; \pm2; \pm3; \ldots$- critical points of the function $f(x)$) .

$4^0$. Using critical points, $x_k = \frac{\pi}{2}(2k+1), k = 0; \pm1; \pm2; \pm3; \ldots$we divide the domain of the function into non-intersecting intervals $(\frac{\pi}{2}(2k+1), \frac{\pi}{2}(2k+3), k = 0; \pm1; \pm2; \pm3; \ldots$. For values $x$ from intervals $(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi), n = 0; \pm1; \pm2; \pm3; \ldots$the inequality holds $f'(x) < 0$, i.e.

$$cosx(2sinx - 5) < 0.$$

For values *of* $x$ from intervals $(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi), n = 0; \pm1; \pm2; \pm3; \ldots$the inequality $f'(x) < 0$holds . That is,

$$cosx(2sinx - 5) > 0.$$

Therefore, on intervals $(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi), n = 0; \pm1; \pm2; \pm3; \ldots$the function $f(x)$decreases, and on intervals

$(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi), n = 0; \pm1; \pm2; \pm3; \ldots$is increasing.

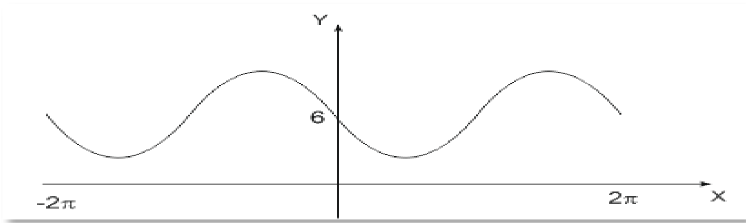
$5^0$. Let us designate the critical points on the number axis and, using the signs of the derivative $f'(x)$when passing through these critical points, determine whether they are extremum points.


$6^0$. Let's find the extrema of the function $f(x)$by calculating the values of the function $f(x)$at the points of maximum and minimum.

$$f\left(-\frac{\pi}{2} + 2n\pi\right) = sin^2\left(-\frac{\pi}{2} + 2n\pi\right) - 5sin\left(-\frac{\pi}{2} + 2n\pi\right) + 6 = 1 + 5 + 6 = 12,$$

$$f\left(\frac{\pi}{2} + 2n\pi\right) = sin^2\left(\frac{\pi}{2} + 2n\pi\right) - 5sin\left(\frac{\pi}{2} + 2n\pi\right) + 6 = 1 - 5 + 6 = 2,$$

$$n = 0; \pm1; \pm2; \pm3; \ldots$$




*Example 3.* Investigate the monotonicity and extremum of a function.

$$f(x) = x\sqrt{1 - x^2}.$$

*Solution* : Let's conduct research on the points of the proposed scheme:

$1^0$. The given function is defined on the segment$[-1,1]$.

$2^0$. We determine the derivative of the given function:

$$f'(x) = \sqrt{1 - x^2} + \frac{x(-2x)}{2\sqrt{1 - x^2}} = \frac{1 - 2x^2}{\sqrt{1 - x^2}}, (x \in (-1,1)).$$

$3^0$. The critical points of a function $f(x)$from its domain of definition are the points $x_1 = -\frac{1}{\sqrt{2}}$, $x_2 = \frac{1}{\sqrt{2}}$, since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$$f'(x_1) = f'(x_2) = 0.$$

4 $^0$. Using the critical points found, we divide the domain of the function into non-intersecting intervals $\left(-1, -\frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 1\right)$.. At $x \in \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$the inequality is true $f'(x) > 0$. When $x \in \left(-1, -\frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right)$the inequality $f'(x) < 0$.is true

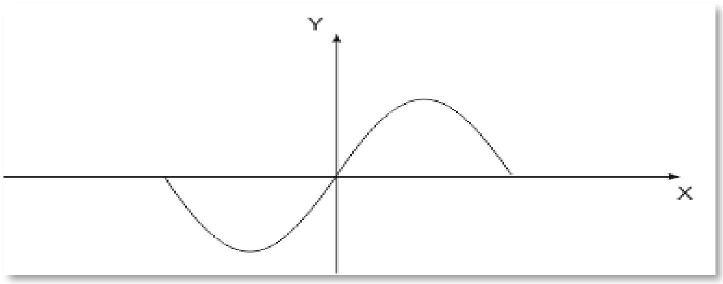
Therefore, on the interval $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$the function increases, $f(x)$decreases over intervals .$\left(-1, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 1\right)$

5 $^0$. Let us designate the critical points on the number axis and, using the signs of the derivative $f'(x)$when passing through these critical points, determine whether they are extremum points.


6 $^0$. Let's find the extrema of the function $f(x)$by calculating the values of the function $f(x)$at the point of maximum $\frac{1}{\sqrt{2}}$, as well as minimum $-\frac{1}{\sqrt{2}}$:

$$f\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}\sqrt{1 - \left(-\frac{1}{\sqrt{2}}\right)^2} = -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = -\frac{1}{2},$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{1}{2}.$$





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