Volume 2, Issue 11, November - 2024 ISSN (E): 2938-3803

EVALUATION OF UNBOUNDED SOLUTIONS OF NON-DIVERGENT NON-LINEAR PARABOLIC EQUATIONS

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Abstract:
This study explores the qualitative properties and unbounded solutions of nonlinear parabolic equations of non-divergent form, focusing on applications such as heat diffusion, biological species diffusion, and combustion processes. The Cauchy problem for these equations is examined, with an emphasis on constructing self-similar solutions and assessing their quality indicators. A theorem is proven establishing a lower solution for the problem under specific conditions. These findings provide preliminary insights into the asymptotics of exact solutions and contribute to the mathematical modeling of nonlinear media with blow-up modes.

Keywords: Nonlinear parabolic equations, non-divergent form, Cauchy problem, self-similar solutions, blow-up solutions, qualitative properties, asymptotic behavior, mathematical modeling, nonlinear media, heat diffusion processes.

Introduction

On a global scale, research dedicated to unbounded solutions of nonlinear parabolic equations of non-divergent form is relevant and necessary. This system of equations is the object of research for such problems as thermal nuclear laser synthesis, heat diffusion processes, and filtration. Therefore, the study of mathematical models in nonlinear media with blow-up mode remains one of the important tasks of mathematics.

Work [1, 2] analyzes the existence and absence of global and blow-up solutions to the Cauchy problem for a non-divergent equation with a source. In a number of works, the asymptotic behavior and qualitative properties of positive solutions to the Cauchy problem for nonlinear equations were studied; [3] the explosion times for nonlinear parabolic systems of non-divergent form were estimated; [4] the existence and absence of global solutions for parabolic systems of non-divergent form with source and variable density were studied, and estimates of solutions were obtained [5].

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Problem statement

Let's consider the Cauchy problem for a non-divergent equation in the domain $Q=(0,T)\times R$:

$$\frac{\partial u}{\partial t}=u^{\frac{3}{2}}\left[\frac{\partial}{\partial x}\left(u^{\frac{1}{2}}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(u^{\frac{1}{2}}\frac{\partial u}{\partial y}\right)\right]+u^2 \qquad (1)$$

$$u\big|_{t=o}=u_0(x)\ge 0,\ x\in R \qquad (2)$$

It is known that equation (1) represents various processes depending on the values of the numerical parameters: biological species diffusion, resistance diffusion phenomena in weak magnetic fields, the spread of infectious diseases, diffusion combustion processes, etc. To numerically solve this type of problem, an automodeled solution is constructed, and the quality indicators of the solution to the problem are studied. The self-similar solutions constructed using the nonlinear separation method can then be used as an initial approximation. We look for the solution in the following form:

$$u(x,t)=\bar{u}(t)\omega(|x|,\tau(t))$$

$$\bar{u}(t)=A(T-t)^{\gamma}$$

here: $\gamma=-1$, $A=1$, $|x|=\sqrt{x^2+y^2}$

$$\tau(t)=-A^2\frac{(T-t)^{2\gamma+1}}{2\gamma+1}, \qquad \omega(\tau,x)=f(\xi), \qquad \xi=|x|\tau^{-\frac{1}{2}}$$

Function f(ξ) is a solution of the following self-similar equation:

$$f^{\frac{3}{2}}\xi^{-1}\frac{d}{d\xi}\left(f^{\frac{1}{2}}\xi\frac{df}{d\xi}\right)+\frac{\xi}{2}\frac{df}{d\xi}+(f^2-f)=0.$$

Main results

To investigate the qualitative properties of equation (1), consider the following function:

$$f(\xi)=B(a+\xi^n)^m$$

in which: $m=\frac{1}{2}$, $n=2$, $a>0$.

$$B=\sqrt{\frac{2}{3}}$$

The theorem has been proven that the found function is a lower solution to problems (1) and (2).

Theorem. $B=\sqrt{\frac{2}{3}}$, $m=\frac{1}{2}$ and let the condition $u\_(t,x)\le u(t,x)$ be satisfied for n = 2. Let the condition ff be satisfied for n = 2. Then, for general solutions to problems (1) and (2) at $t\to T$

Web of Humanities: Journal of Social Science and Humanitarian Research

Volume 2, Issue 11, November - 2024 ISSN (E): 2938-3803

the following estimate holds: $u\_(t,x) \le u(t,x)$, $x \in R$.

here $u\_(t,x) = \sqrt{\frac{2}{3}} \cdot (T-t)^{-1} \cdot \left(a - \xi^2\right)^{\frac{1}{2}}$, $\bar{u}(t), \xi$ – functions defined above. Evaluating or finding the asymptotics of an exact solution is also a good result when you can't find it. The found lower or higher solutions give us preliminary information about the exact solution.

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